

Simulations of Pion Production in a Tantalum Rod Target using GEANT4 with comparison to MARS

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Abstract

Two of the most important future experiments in particle physics are the Neutrino Factory and Muon Collider. Both these complexes require an intense muon beam derived from pion decay. This study investigates the baryonic meson production of a selection of proton beam energies on a solid, cylindrical tantalum target. The pions produced will be examined for overall yield and “capturable” yield. This paper will act as a comparison between the GEANT4 results and MARS results for an identical target setup. The transmission through two different Neutrino Factory front-ends will also be investigated.

UK Neutrino Factory Note 30

Contents

1	Simulation	2
2	Pion yields as a function of incident proton energy	3
2.1	Comparison between GEANT4 hadronic models	5
3	Pion angular distribution	7
4	Pion yield inside 20° transverse acceptance	9
5	Pion yield within 100 MeV transverse momentum and 20° transverse angular acceptance	10
6	Simulation of Pion Yield after Transmission through Neutrino Factory Front-Ends	12
6.1	Hadronic model comparison of transmission	15
7	Conclusion	17
8	Further Work	18

1 Simulation

The simulation was carried out using a program called G4Beamline. G4Beamline is a particle tracking program based on the GEANT4 toolkit [1] and FermiLab Beamtools [2]. The most recent version of G4Beamline was used, which is compiled using Geant4 6.2p02. GEANT4 distributes its physical process models in specific groups called physics use-cases. The use-case that was chosen for this simulation was QGSP although it will be shown in this document that its validity for initial proton kinetic energies of $< 10\text{GeV}$ is questionable and other alternatives are investigated. However QGSP is the most accurate physics use-case above this energy. The analysis tool used is called Histoscope [3] and is again a Fermilab produced package.

The geometry chosen for this simulation was identical to that chosen in the MARS simulation [4]. In this simulation, a parallel, parabolic proton beam with no longitudinal or momentum spread is incident perpendicular to the upstream edge of a solid tantalum cylinder of dimensions 20cm along beam axis and 1 cm radius. The tantalum rod was surrounded by a detector volume of vacuum protruding to a distance of 1mm around every surface of the rod. The beam was started at exactly the upstream end of the rod. All particles crossing the idealised detector volume were recorded. The constructed geometry can be seen in Figure 1.

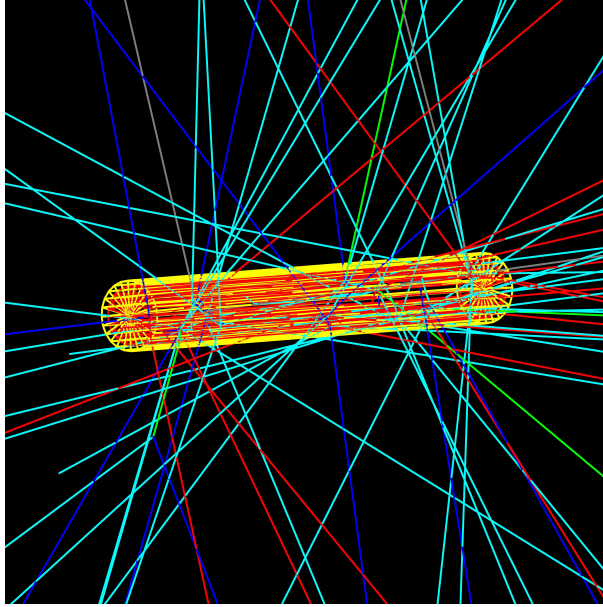


Figure 1: Open GL visualisation of the target geometry. The target is coloured red with the virtual detector volume coloured yellow. Protons are red, π^+ green, π^- grey, μ^+ blue and all neutral particles are coloured cyan (almost all are neutrons).

G4beamline accepts four different types of beam: Gaussian, rectangular, ascii and histo. The ascii beam input was chosen for this simulation. Production of the parabolically distributed proton beam was implemented by a C++ program created by the author. This program outputs a beam in the relevant ascii format for G4Beamline to read. The program is fairly flexible and will provide a parabolic beam scaled to any size, with any number of events of any particle of any uniform momentum. This program is available from the author. The parabolic beam has the distribution $f(r) \propto 1-r^2$ with r the scaled radius of the desired beam. Each Monte Carlo run consisted of 100k protons on target with each job submitted to the Scotgrid computer farm to process. Proton beams with kinetic energies in the range $2.2\text{GeV} \rightarrow 120\text{GeV}$ will be studied. This range covers proposed future proton drivers envisaged for a Neutrino Factory (Table 1).

Proton Energy (GeV)	Proton Momentum (GeV/c)	Proton Driver
2.2	3.0	CERN SPL
3	3.8	RAL ISIS upgrade low energy ring
4	4.8	Possible highest energy for SPL
5	5.9	RAL Study for green field site
6	6.9	RAL ISIS upgrade high energy ring (6-8GeV)
8	8.9	Fermilab driver study 2
10	10.9	
15	15.9	RAL study for machine in the CERN ISR tunnel, Fermilab driver study 1 (16GeV)
20	20.9	Brookhaven AGS upgrade (24GeV)
30	30.9	JPARC high energy ring initial energy, RAL study for a CERN PS replacement in the ISR tunnel
40	40.9	
50	50.9	JPARC high energy ring ultimate energy
75	75.9	
100	100.9	
120	120.9	Fermilab main injector, currently in use for NuMI beam

Table 1: Incident proton energies used in simulations [4]

2 Pion yields as a function of incident proton energy

The first quantity studied is the total number of pions produced by the tantalum target. This makes no provision for investigating the pion phase space volume that can be captured by the downstream optics. This will be partially dealt with in later sections. The absolute numbers of π^+ and π^- produced are not a particularly useful quantity to study when looking for the optimal proton energy for a Neutrino Factory. A more appropriate quantity is pions per proton normalised to the proton beam energy as this is proportional to number of pions produced per Watt of beam power.

The hadronic physics model that is used is QGSP. This is based on a Quark-Gluon String model and is the recommended GEANT4 physics use-case for the majority of the energy range detailed in this document. For energies $< 25\text{GeV}$ the GHEISHA model is used and for $E > 25\text{GeV}$ the Quark-Gluon String model is adopted. The quark gluon string model is used for initial projectile-nucleus collisions with cross-sections for string excitations calculated. The behaviour of the nucleus after the initial violent collision is handled by a pre-equilibrium decay model [5].

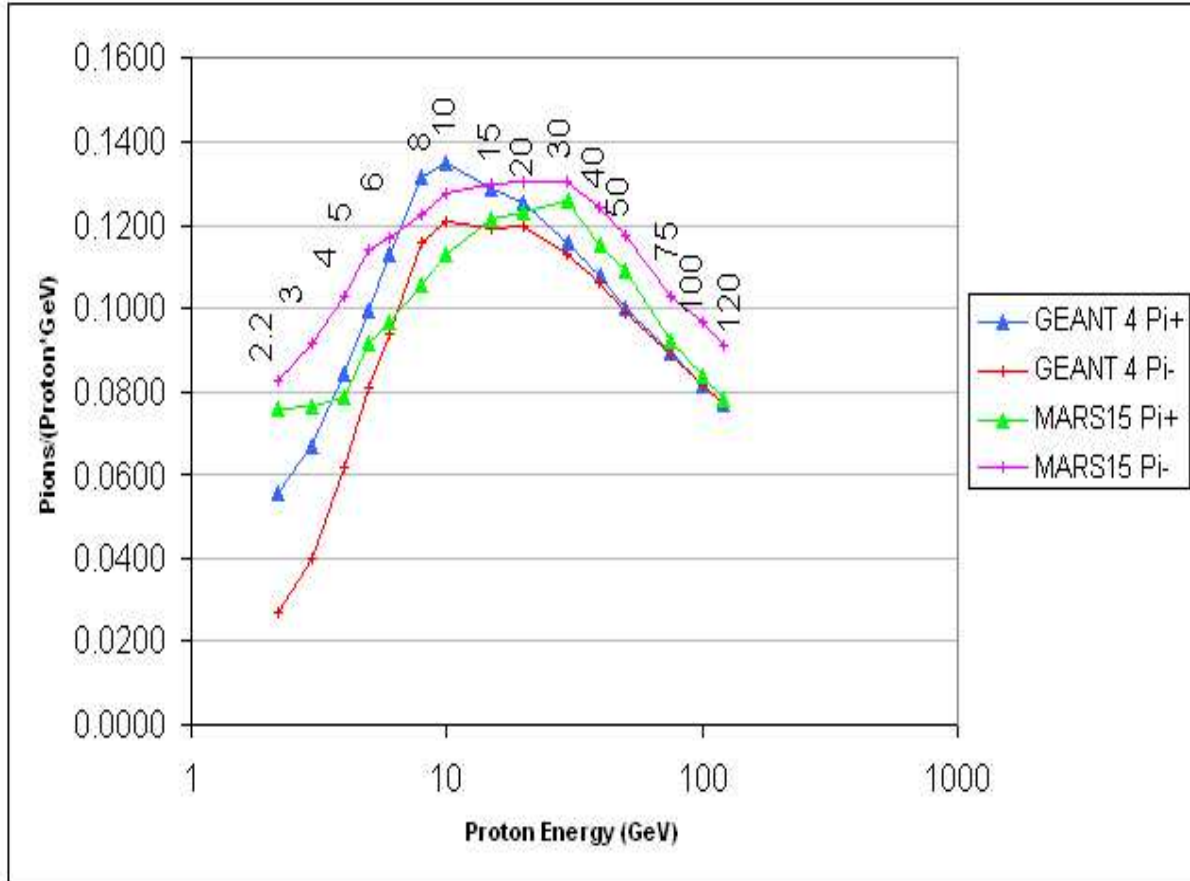


Figure 2: Total π^\pm yields

QGSP was used to produce the results shown in Figure 2. Also shown in this plot are the results of a MARS simulation with an identical beam and target geometry [4]¹⁾. The GEANT4 results clearly show that over the whole range, the number of π^+ produced per proton on target is always greater than the number of π^- produced. The MARS data shows the opposite. The variation in discrepancy between the two models can be seen in Figure 3. The highest excess of π^+ in GEANT4 over MARS is at 8GeV where there is a 19.6% difference. The highest excess in π^+ in MARS over GEANT4 is at 2.2GeV where the difference is 36.8%. The difference between models is in the $\pm 8\%$ range for proton energies of $\geq 15\text{GeV}$. The disagreement between both codes is greater when looking at π^- . For energies of $\leq 6\text{GeV}$ it is particularly bad with differences of 24% \rightarrow 200%

¹⁾Note that throughout this document when referring to MARS data its source is [4]

the later value associated with a proton energy of 2.2GeV. For energies $>6\text{GeV}$ model agreement is better although still at the 8% \rightarrow 18% level and the GEANT4 values are consistently below those of MARS.

The results suggest that the optimal proton energy for a neutrino factory is in the range 8-15GeV. It should also be noted that the discrepancy between GEANT4 and MARS decreases as one moves closer to the upper limit of this range.

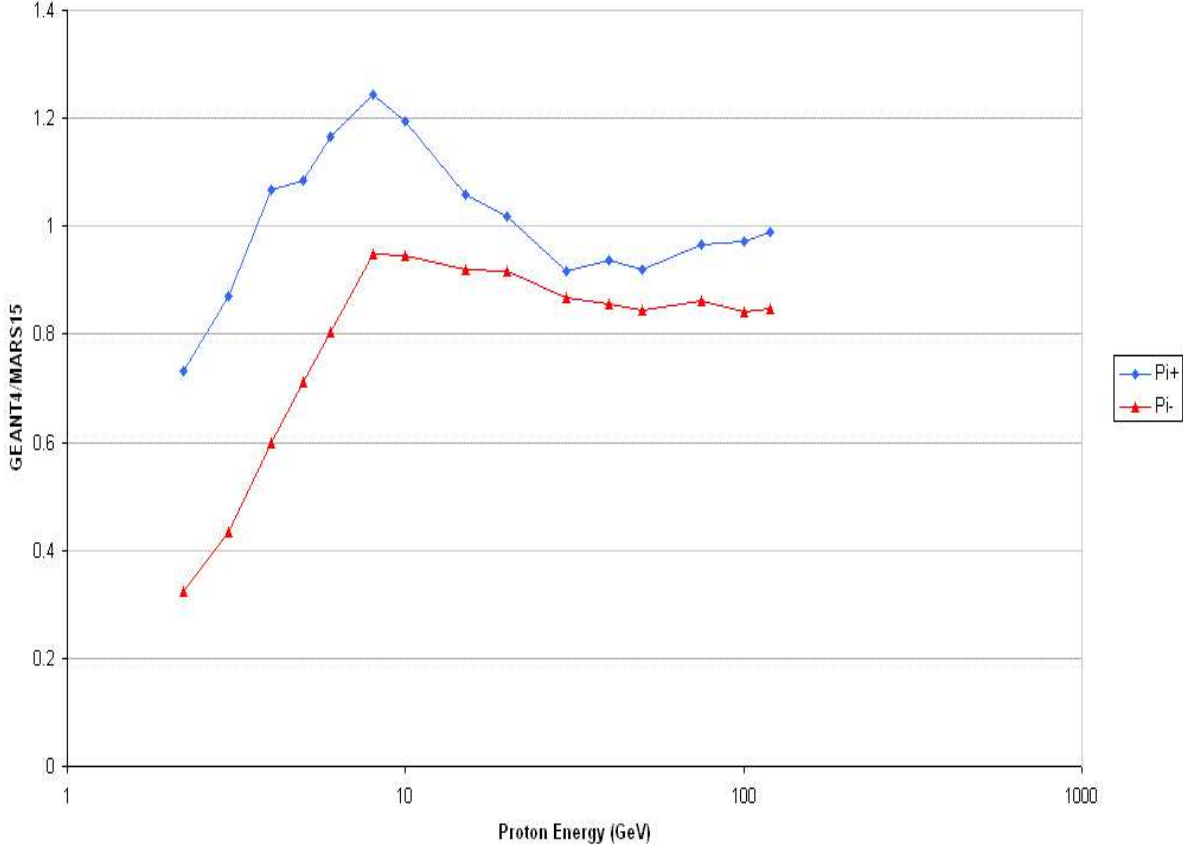


Figure 3: Pion per proton*Energy(GeV)variations between GEANT4 and MARS

2.1 Comparison between GEANT4 hadronic models

The region between 1-10 GeV is a grey region in GEANT4 hadronic physics. This energy regime has been probed by the HARP experiment [6] specifically to better understand pion production for a Neutrino Factory. Results from HARP should be released shortly and will help remedy our poor understanding. It is in this kinematic region that the models describing lower energy hadronic physics e.g. Binary Cascade and Bertini Cascade are at the upper limit of their validity range ($10\text{MeV} \rightarrow 8\text{GeV}$). On the other hand the quark-gluon string model which deals with higher energy hadronic physics is valid in the range $5\text{GeV} \rightarrow \text{TeV}$ is at the lower edge of its validity. By comparison, the MARS15 code uses two hadron production models. For $E < 5\text{GeV}$ MARS uses the “Cascade-Exciton

Model” CEM2003 algorithms and for energies $> 3\text{GeV}$ an “inclusive hadron production” model is used.

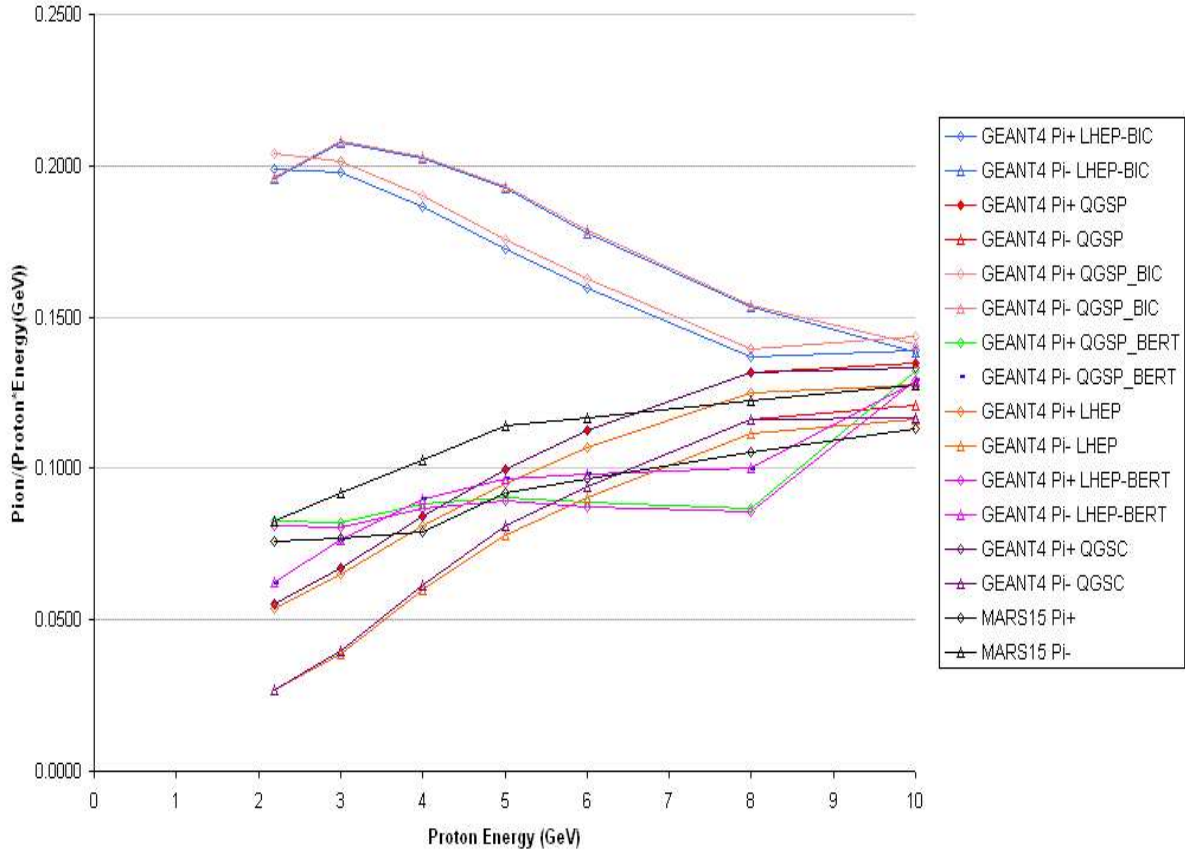


Figure 4: Comparison between GEANT4 physics use cases in the range 2.2-10GeV

This uncertainty prompted a study of each GEANT4 physics use-case that has at least part of its range of validity between $1 \rightarrow 10$ GeV. The disagreement in π^\pm GeV yields is considerable and evident in the results shown in the previous section. Further simulations were computed altering the physics used to calculate the hadron interactions in the target. A summary of the different physics use cases studied and a brief mention of specific physics models used can be seen in Table 2.

The results of this comparison are shown in Figure 4 with higher resolution views of the higher and lower yield models shown in Appendix A. It should be noted that certain points are indistinguishable as they lie on top of each other. These include the points for QGSP and QGSC which differ only at 10GeV and LHEP-BERT and QGSP-BERT. The first point to comment on is the distinct variation between models particularly at low proton energies. The variation between the binary cascade models and others is of the order 100% at low energies. The ratio of π^+ to π^- also varies between models: QGSP, QGSC and LHEP all produce more π^+ than π^- whereas both Binary and Bertini Cascade models produce more π^- than π^+ for most of the range. The Bertini models show a large rise between 8-10GeV where the π^+/π^- becomes > 1 . The agreement between all models including MARS15 is significantly better around the 10 GeV mark.

GEANT 4 Use Case	Model implemented
LHEP	GHEISHA inherited from G3
LHEP-BERT	E < 3GeV Bertini cascade, E > 3GeV GHEISHA
LHEP-BIC	LHEP-BIC E < 3GeV Binary cascade, E > 3GeV GHEISHA
QGSP	E < 25GeV GHEISHA, E > 25GeV quark-gluon string model
QGSP-BERT	QGSP-BERT E < 3GeV Bertini cascade, 3 < E < 25GeV GHEISHA, E > 25 GeV quark gluon string model
QGSP-BIC	E < 3GeV Binary cascade, 3 < E < 25GeV GHEISHA, E > 25 quark gluon string model
QGSC	E < 25GeV GHEISHA, E > 25GeV quark-gluon string model, chiral invariance

Table 2: GEANT 4 Hadronic Physics Summary

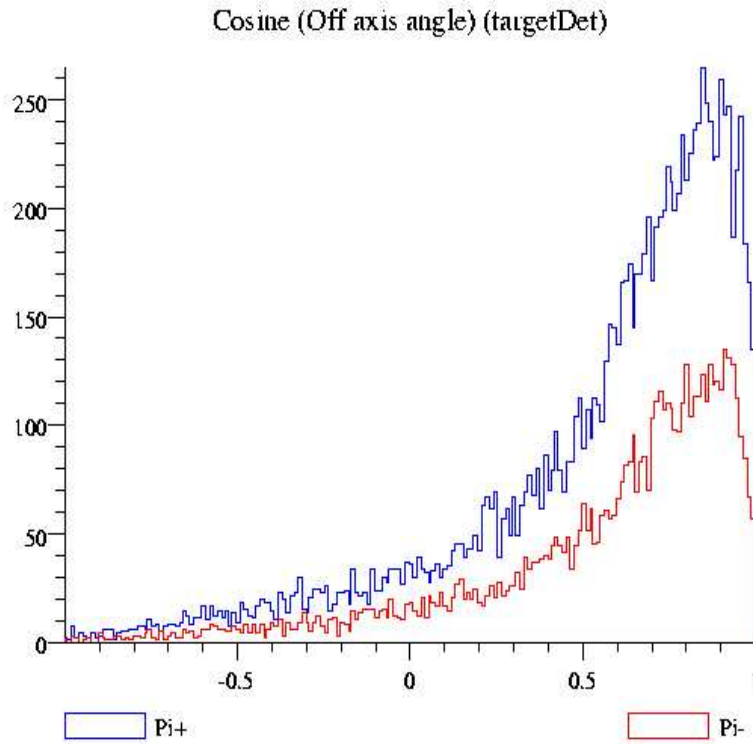


Figure 5: Pion angular distribution with incident proton energy of 2.2GeV. X-axis is the Cosine of the angle made between the outgoing pion and the z-axis of the target

3 Pion angular distribution

In producing a Neutrino Factory target one is obviously interested in the π^\pm angular distributions. In this section, the off-axis angle that the outgoing pion makes with the z axis of the target will be studied. The pion distributions calculated using QGSP created by 2.2 and 15 GeV incident proton beams can be seen in Figures 5 and 6 respectively. Both figures have 0.01745 radian bins. It can be seen from close inspection of both plots that outgoing π^+ are forward focussed to a similar degree when compared with outgoing

π^- . It is also evident that as the energy increases the focussing of both π^\pm increases. This is to be expected. Note that obviously those pions sitting in bins less than $\cos\theta = 0$ will be backward travelling and as such are uncapturable as are those with high exit angle (see later).

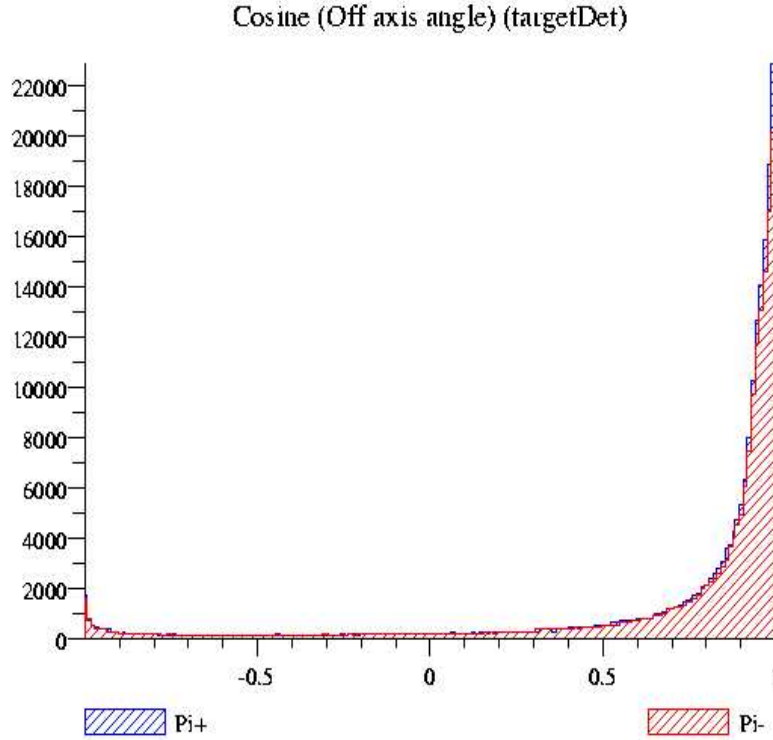


Figure 6: Pion angular distribution with incident proton energy of 15 GeV. X-axis is the cosine of the angle made between the outgoing pion and the z-axis of the target

In Figure 7 the pion angular distributions are displayed where the angles that contained 75%, 50% and 25% of the pions are plotted for both π^\pm . Also shown are the same values obtained by MARS. Firstly commenting on purely the GEANT4 results it can be seen that generally both π^\pm are equally focussed with differences of $< 10\%$. Also there is a peak in the distribution from 20-30 GeV. This is likely to be partially if not wholly artificial as there is a QGSP model transition region at 25 GeV. The effect of this is felt more strongly for those pions at highly transverse exit velocities with the effect on the more focussed π^\pm much less. This makes sense as the reaction cross section for pion production is less well known at high transverse angles than those forward focussed angles, hence both models will struggle to cope with these particles at the transition region.

Comparing the GEANT4 results with the MARS data it can be seen that in general the π^\pm are significantly more forward focussed in GEANT4. The MARS π^\pm approach the GEANT4 results for high initial proton energies, in particular in the more forward focussed direction. The π^+ produced by MARS are closer to the GEANT4 results compared with the MARS π^- across the whole angular range.

The final notable feature is that the large dip in the MARS data between 2.2 GeV and 6 GeV does look from these results to be non-physical and a remnant of the MARS model transition in this region. By comparison the GEANT4 results show a more gradual

Pions angular distribution

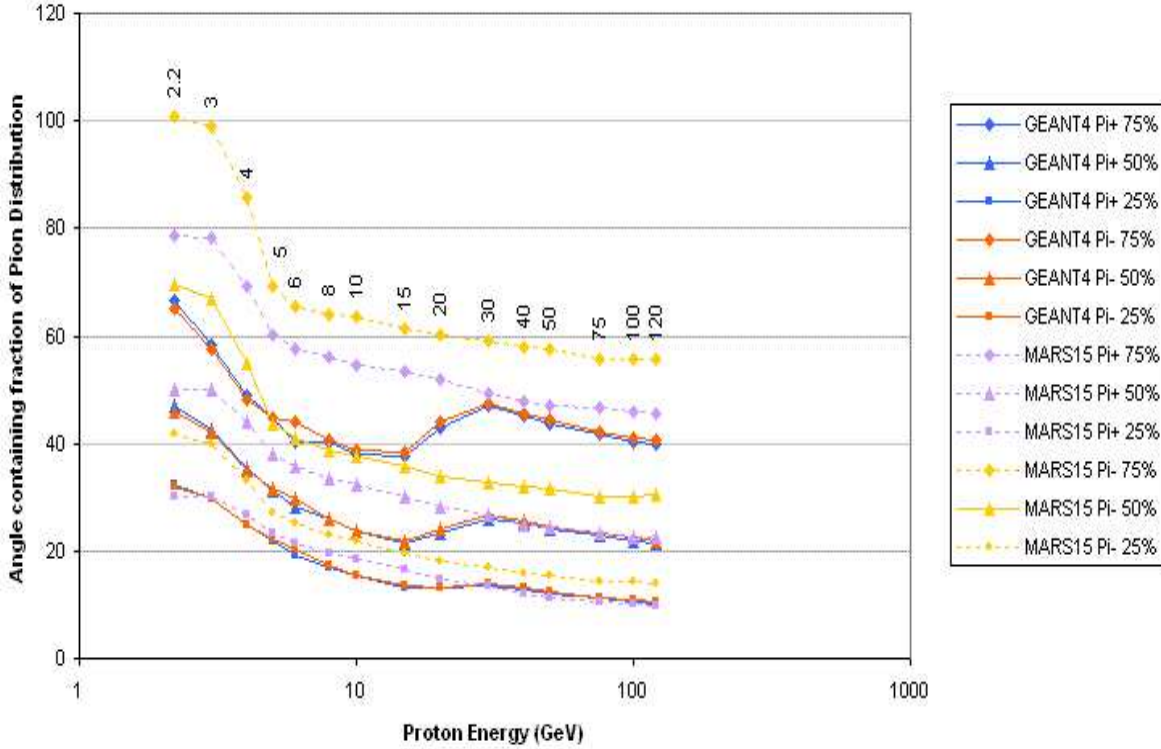


Figure 7: Plot of angles containing 25%, 50% and 75% of pion distribution

and arguably more believable result.

4 Pion yield inside 20° transverse acceptance

In constructing a target applicable to a Neutrino Factory obviously the most important aspect of pion production to study is the number of “capturable” pions produced per unit beam power. In an approximate attempt to investigate this, a cut was imposed to the data shown previously. This cut eliminates pions whose exit momentum vector lies within a 20° cone pointing downstream centred along beam axis. Pions lying outside this volume of phase space will be unlikely to be captured by downstream optics.

The results can be seen in Figure 8. The most obvious feature is the peak in the GEANT4 data at 15GeV for both π^\pm . The QGSP model transition region at 25GeV can be clearly seen in the sharp change of behavior from 30GeV onwards. This peak may not be physical and a further comparison of different physics use-cases should be carried out in the future. The MARS data differs significantly in the peak region with better agreement at lower energies (especially 3-5GeV) and higher energies. Pion yields within the 20° cone are greater in GEANT4 than MARS15 over the range 4-30 GeV although this does include the GEANT4 peak region. It is evident that in this range QGSP calculates pions produced to be more forwardly directed than MARS. Purely based on the GEANT4

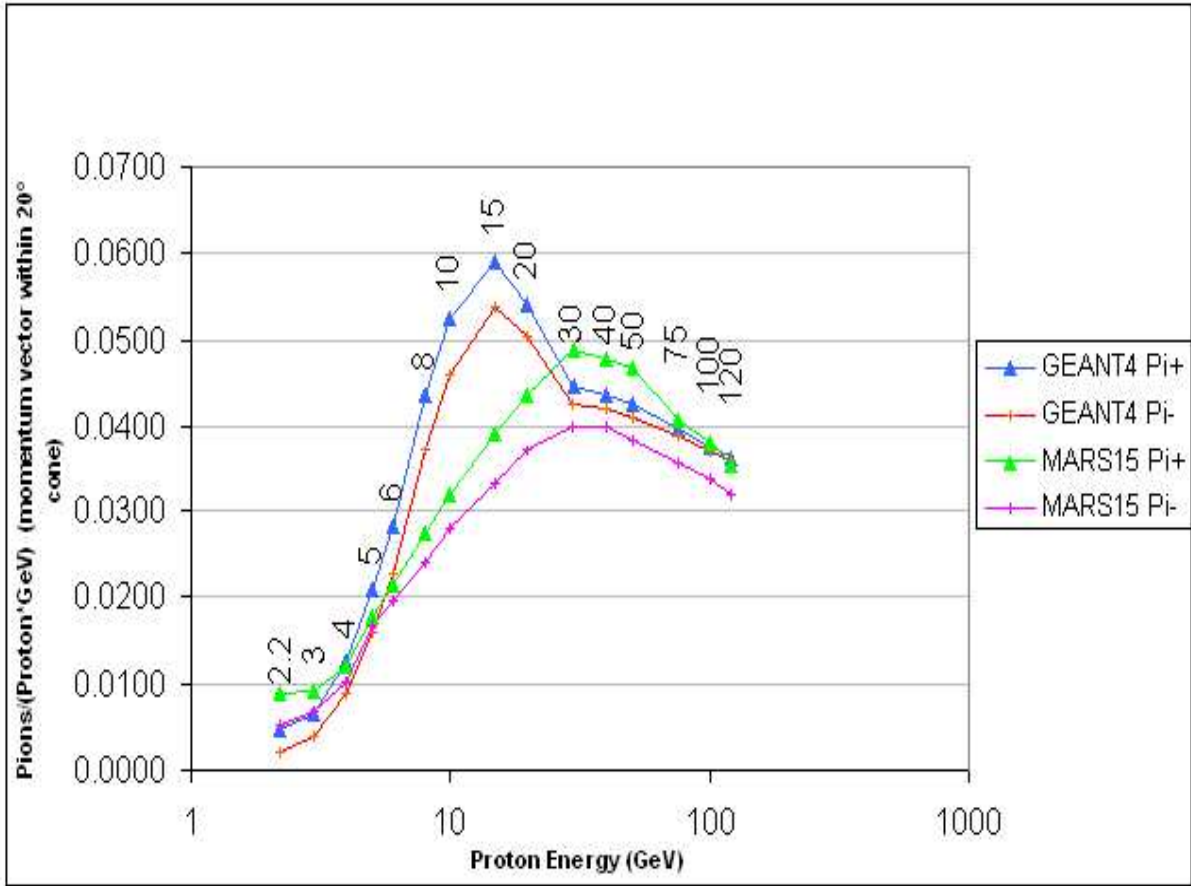


Figure 8: Pion per (Proton*Energy(GeV)) within 20° cone

results (with a caveat on the model accuracy) a proton energy of around 15GeV looks optimal for a Neutrino Factory.

5 Pion yield within 100 MeV transverse momentum and 20° transverse angular acceptance

In the previous section, an angular cut was imposed to eliminate large numbers of pions that do not have the correct kinematics to be captured by the pion decay channel optics i.e. those pions that are backward travelling or that have highly transverse exit angles. A more effective cut on P_t can be made as a population of high momentum pions with high exit angles within the 20° cone will be moving too quickly to be captured. Previous studies [7] have shown that a $P_t < 100\text{MeV}$ cut would be acceptable to study the transmission into the downstream optics. The results of this cut can be seen in Figure 9. The peak mentioned in the previous section is again prevalent as is the model transition region at 25 GeV. The peak is more distinct in this plot than in the previous one, the decrease in relative pion yield for high energies accounting for this. The P_t cut has eliminated the fast, highly transverse phase space pions. These pions are uncapturable and are more numerous with increasing incident proton energy. It is interesting to note that the relative

pion yields in the range 2.2-20GeV have remained the same after the additional Pt cut. This implies that these pions are indeed transversely focussed. The MARS data shows a similar decrease in yields at higher energies. The final point to note is the almost uniform relative shift in pion yields between GEANT4 and MARS for energies >30GeV suggesting that the QGSP algorithms focus pions more in the forward direction than the MARS algorithms. This data (again with the caveat on model accuracy) again suggests the optimal proton energy for a Neutrino Factory is in the range 10-15GeV.

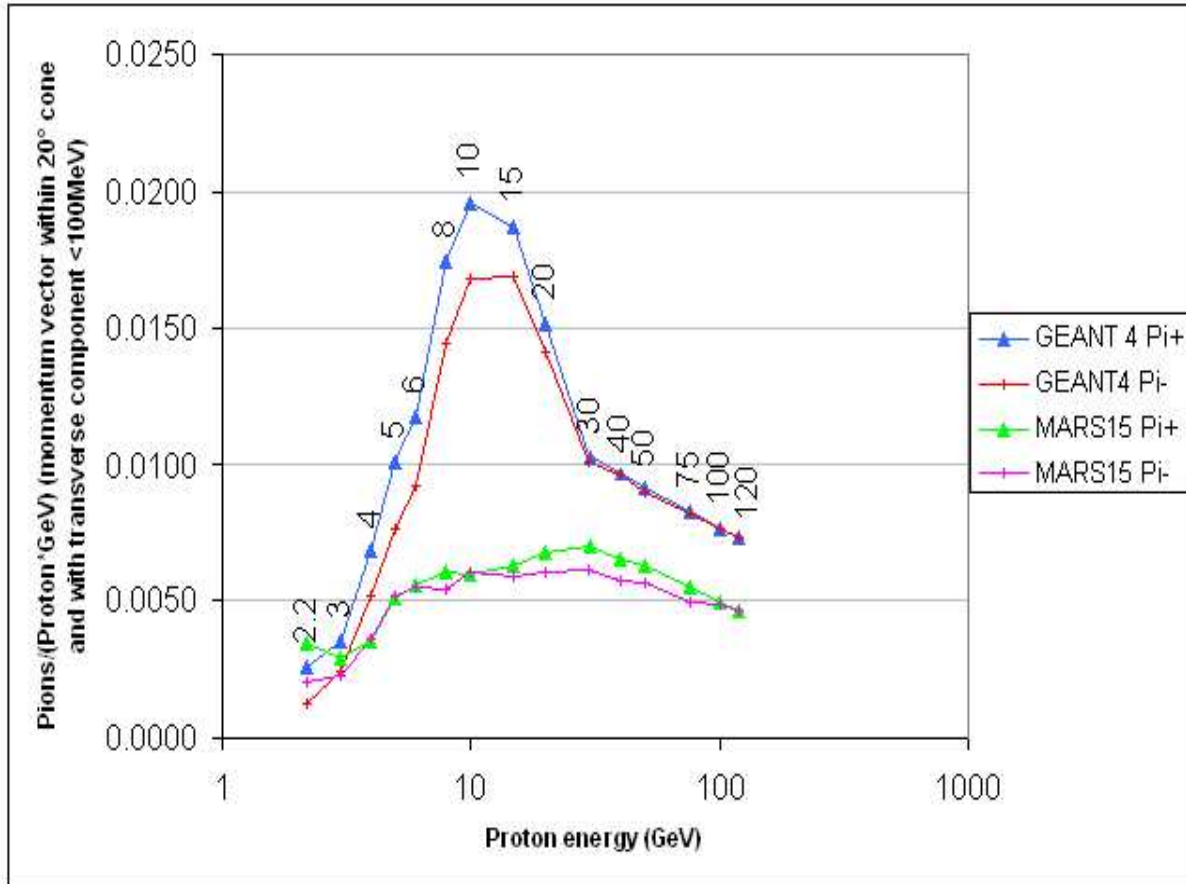


Figure 9: Pion per (Proton*Energy(GeV)) within 20° cone and Pt<100 MeV

6 Simulation of Pion Yield after Transmission through Neutrino Factory Front-Ends

The $\pi^\pm \rightarrow \mu^\pm$ decay in a neutrino factory happens in the muon front-end of the channel. This section will investigate the pion yields after transmission through two different designs of neutrino factory front-ends, namely the Chicane/Linac and Phase Rotation schemes.

In the Chicane/Linac scheme the target is surrounded by a 20T solenoid followed by a solenoidal decay channel consisting of 32 transversely focussing solenoids. The lack of longitudinal focussing leads to longitudinal bunch length stretching and ultimately a debunched muon beam. Longitudinal compression is then achieved by a reverse-phase-slip achromatic lattice which is referred to here as a bending chicane. The chicane consists of four identical sets of dipole triplets with fields designed such that a pencil poly-chromatic beam will emerge from a triplet as a dispersed beam rotated by 101° . This dispersed beam then enters a second triplet and focussed to a pencil beam. This is repeated in the remaining triplets with the bend in the opposite direction. Path-length differences cause recompression longitudinally. The bunched muon beam is then passed through an 88MHz muon linac which accelerates to $400 \pm 100\text{MeV}$

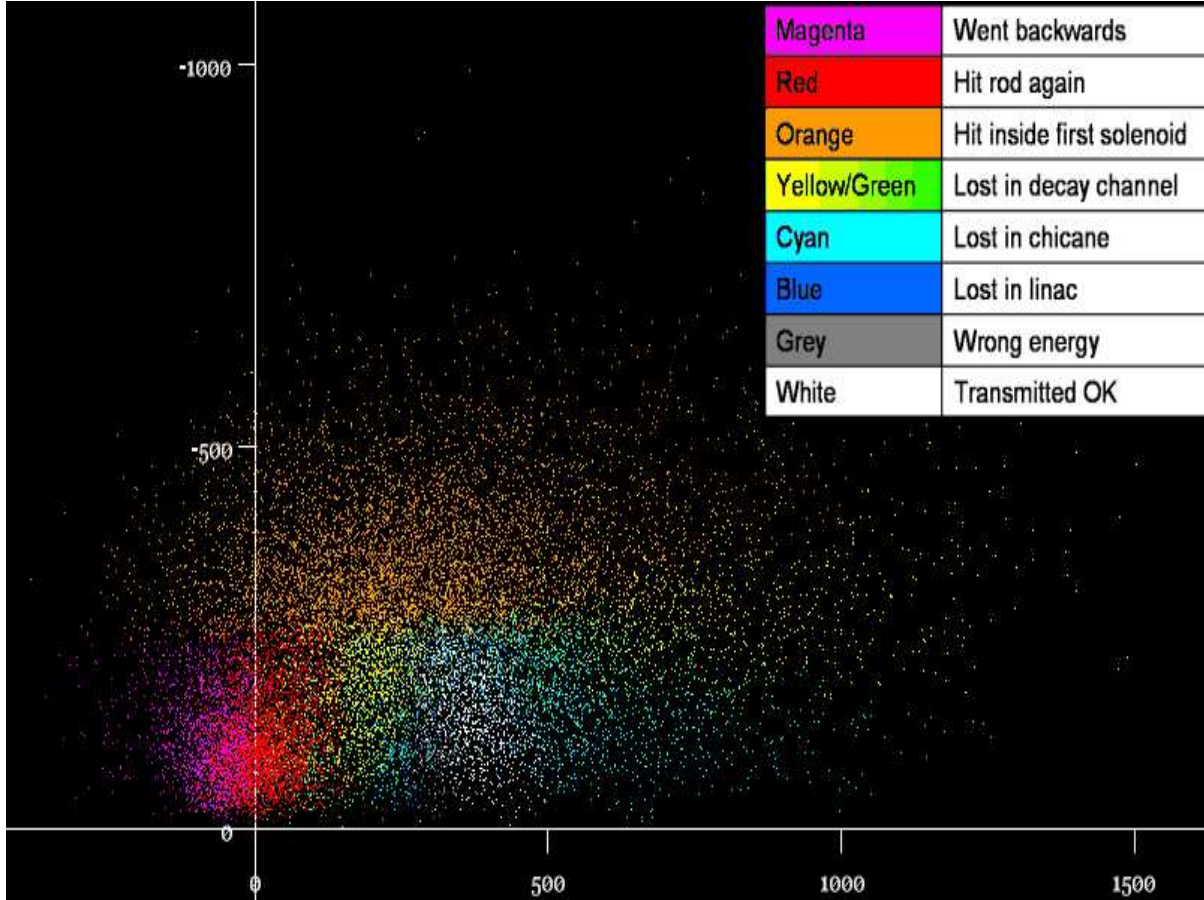


Figure 10: π^+/π^- fate along 2.2GeV optimised Chicane/Linac Channel, P_{long} vs P_{transv} [8]

The Phase-Rotation scheme involves continuing the solenoid channel before using 31.4 MHz RF-cavities to reduce the energy spread with a goal of 180 ± 23 MeV. This muon beam would then be injected into a cooling ring for reduction of normalised emittance.

Particles were tracked through both these front ends using a code called Muon1 [7] that in addition to tracking deals with particle decays. The fate of the particles (i.e. whether they made it through the channel and if not where along the channel they were lost) can be seen for the Phase Rotation scheme (Figure 11) and Chicane/Linac (Figure 10). Both channels were tuned for 2.2 GeV. The distributions were sampled for both schemes after the Linac and RF-Cavities for Chicane/Linac and Phase Rotator schemes respectively. The distributions were binned into two dimensional P_{long} P_{transv} bins of 30 MeV/c and weighted into a probability map.

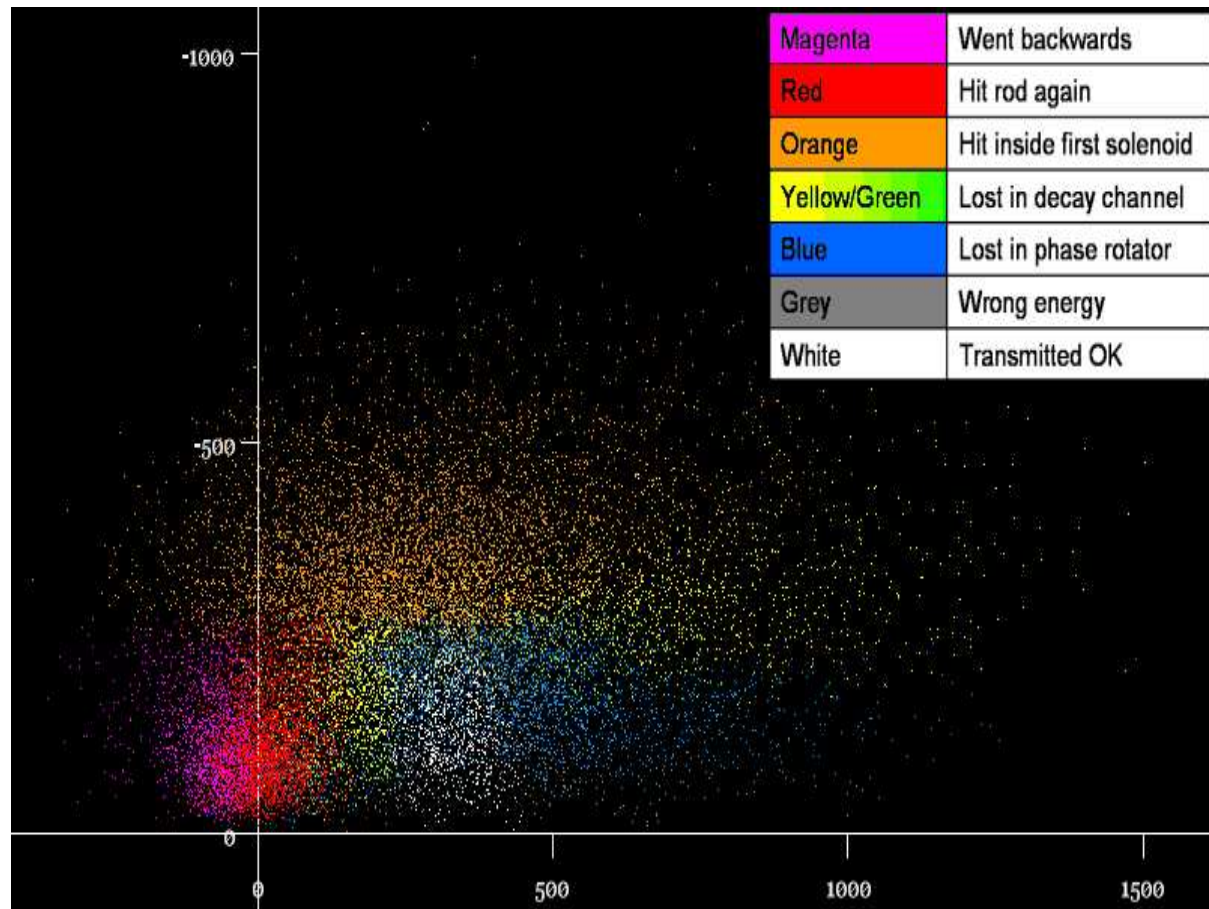


Figure 11: π^+/π^- fate along 2.2 GeV optimised Phase Rotator Channel, P_{long} vs P_{transv} [8]

A program was constructed called ProbabilityMap²⁾ which has functionality such that it could bind with histoscope (See §1) output and calculate weighted transmissions for any particle of any momentum in variable bin sizes for any probability map constructed for any channel. This program was then run on the GEANT4 datasets previously produced using the QGSP physics use-case in §2 for each of the relevant proton driver energies (Table 1). This was repeated for both Chicane/Linac and Phase Rotator schemes and the

²⁾ Available from author

results can be seen in Figures 12, 13. This method is very quick and allows easy estimation of transmission along different channels. Currently only the 2.2GeV channel has been optimised however it will be very easy to run these datasets using other optimisations when they become available.

The results for the number of pions per unit beam power for the Chicane Linac channel (Figure 12) show that more π^+ than π^- per proton beam power are transported until 30GeV where they are transported in equal measures. The peak appears to be around 8GeV although the dip which appears for both π^\pm at 6GeV seems strange and probably non-physical. Negative pions are preferentially transported along the channel at the very highest energies.

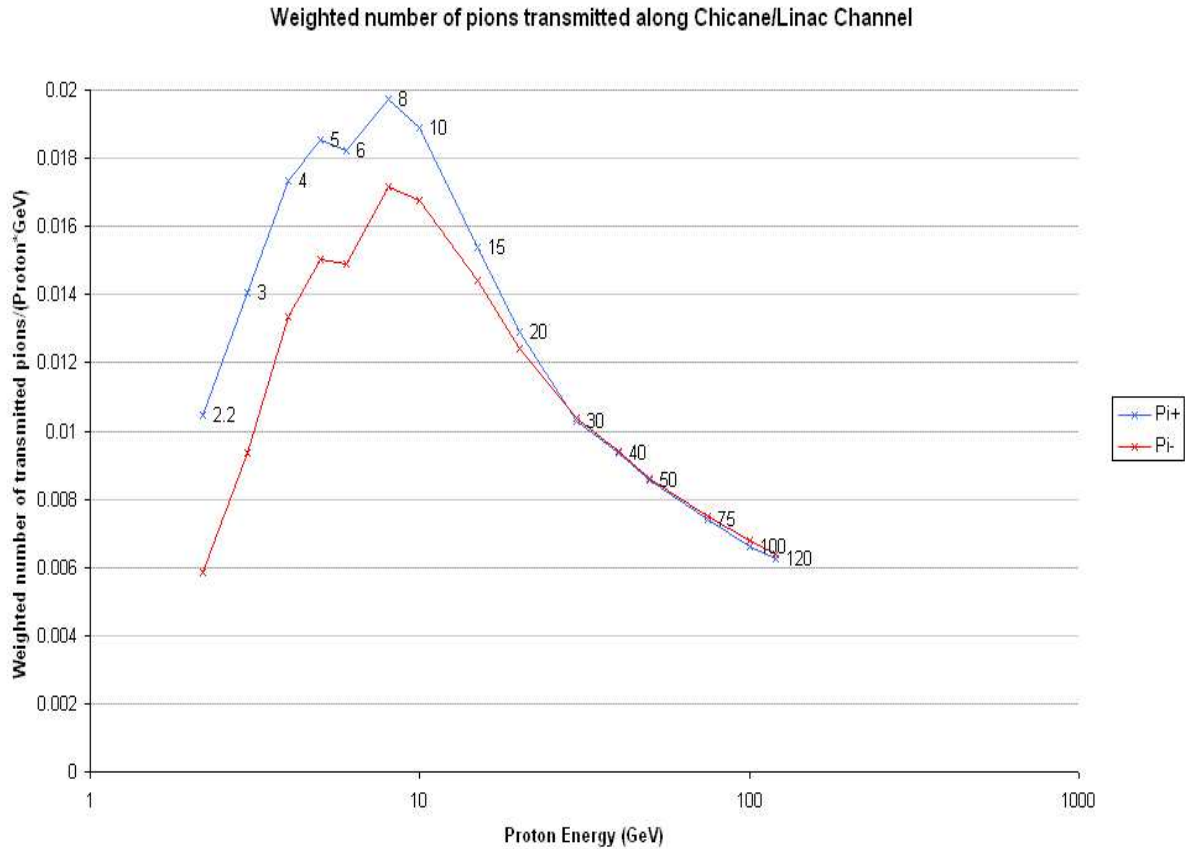


Figure 12: π^+/π^- transmission along 2.2GeV optimised Chicane/Linac Channel

The results for the Phase Rotator channel are shown in Figure 13. It can be seen that many of the features of the Chicane/Linac plot are evident here also. Namely, the peak at 8 GeV, the dip at 6 GeV, the excess of π^+ transmitted compared to π^- until 30GeV and the behavior at higher energies. It can be seen that the number of both pion charge conjugates transmitted differ by only a few percent between schemes at 2.2 GeV. This is not surprising as it is on momentum. On the other hand at the 8 GeV peak the Phase Rotation scheme only transmits around 90% of the pion transmission achieved by the Chicane Linac scheme.

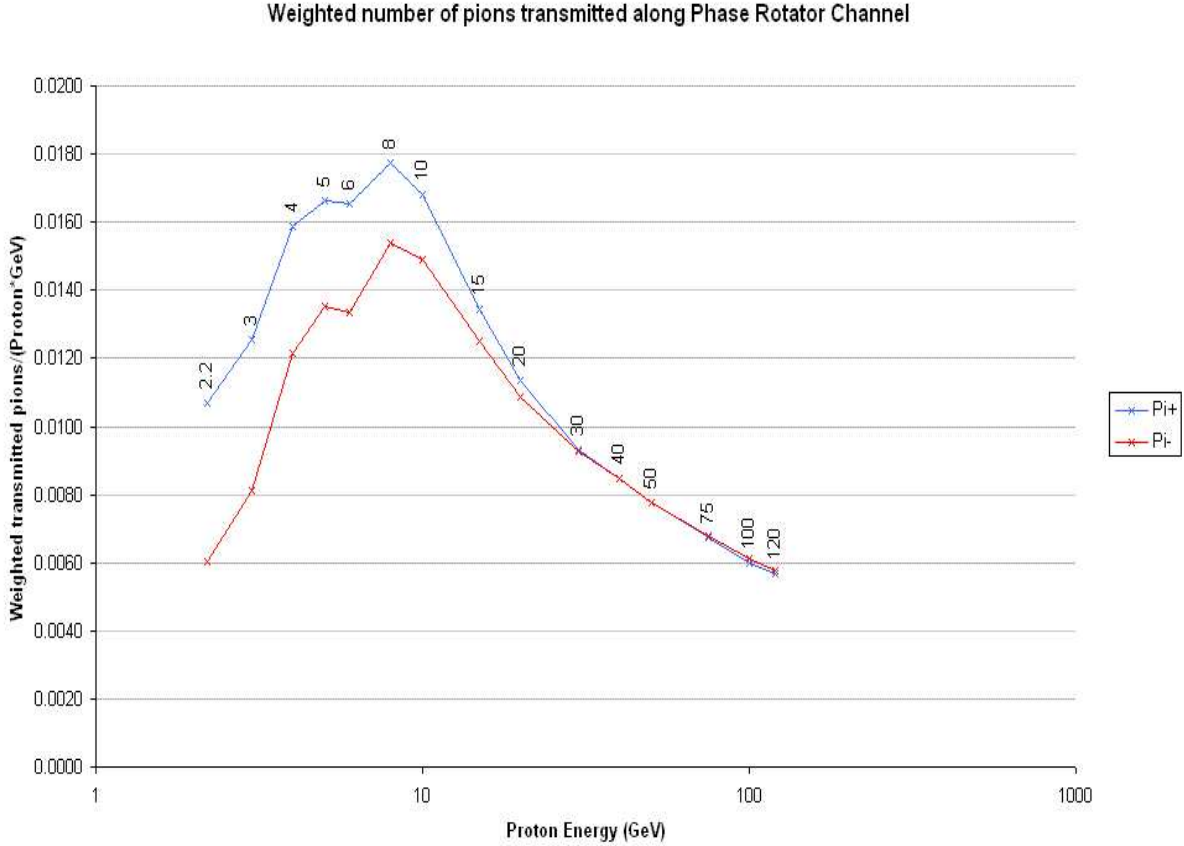


Figure 13: π^+/π^- transmission along 2.2GeV optimised Phase Rotation Channel

6.1 Hadronic model comparison of transmission

The program ProbabilityMap was also run on the data-sets produced using the different GEANT4 use-cases detailed in §2. Both the Chicane/Linac and Phase Rotator maps were used to calculate the transmission for the respective channels.

The transmission results for the Chicane/Linac scheme can be seen in Figure 14. It is noticeable that the binary cascade models have a higher transmission of pions although this is to be expected as these models produced the largest raw pion yields (§2.1). They also predict a higher transmission at lower energies which is also the energy domain with the largest raw yield. The LHEP and QGSP models also perform as the raw pions yields in §2 would suggest with a peak around 8GeV. It is interesting to note that the transmission of the data sets produced by the bertini cascade models has little or no energy dependance in the range $2.2\text{GeV} \rightarrow 8\text{GeV}$. The sharp rise between 8 GeV and 10 GeV is likely due to a model transition region.

The transmission results for the phase rotator channel can be seen in Figure 15. The trends in the data in this plot are similar to the chicane/linac channel. One key difference is that across almost the entire energy scale studied the phase rotator transmission is lower than the chicane/linac equivalent for all non-bertini cascade models. Figure 16 shows the percentage change in transmission between the both optical channels (per proton*GeV) normalised to the chicane/linac channel for the hadronic models discussed.

Positive values are in favour of the non-cooling chicane/linac channel, negative values in favour of the phase-rotator scheme. It can be seen that at 2.2 GeV the QGSP and LHEP π^\pm transmission for the phase rotation channel is between 2-3% greater than that of the chicane/linac channel. For energies greater than this the difference is between 8-13% in favour of the chicane/linac channel. The transmission of pions created by binary cascade models is consistently greater for the chicane/linac scheme, differences being in the range 8% \rightarrow 10%. The transmission of pions created by the bertini cascade models for both schemes are shown to agree to roughly 5% across the entire energy range studied. The phase rotator channel performs better across the majority of proton energies with the only exception being at 10 GeV where the difference is approximately 10% in favour of the chicane/linac scheme.

These results suggest that the chicane/linac channel is better at transmitting off momentum π^+ and π^- than the phase rotator channel. This is the case for all non-Bertini Cascade hadron production models. These results are of course preliminary. A more rigorous comparison between transmission at different proton driver energies and between both channels at those energies can only be conducted when those channels are optimised to the relevant momentum magnetic lattices.

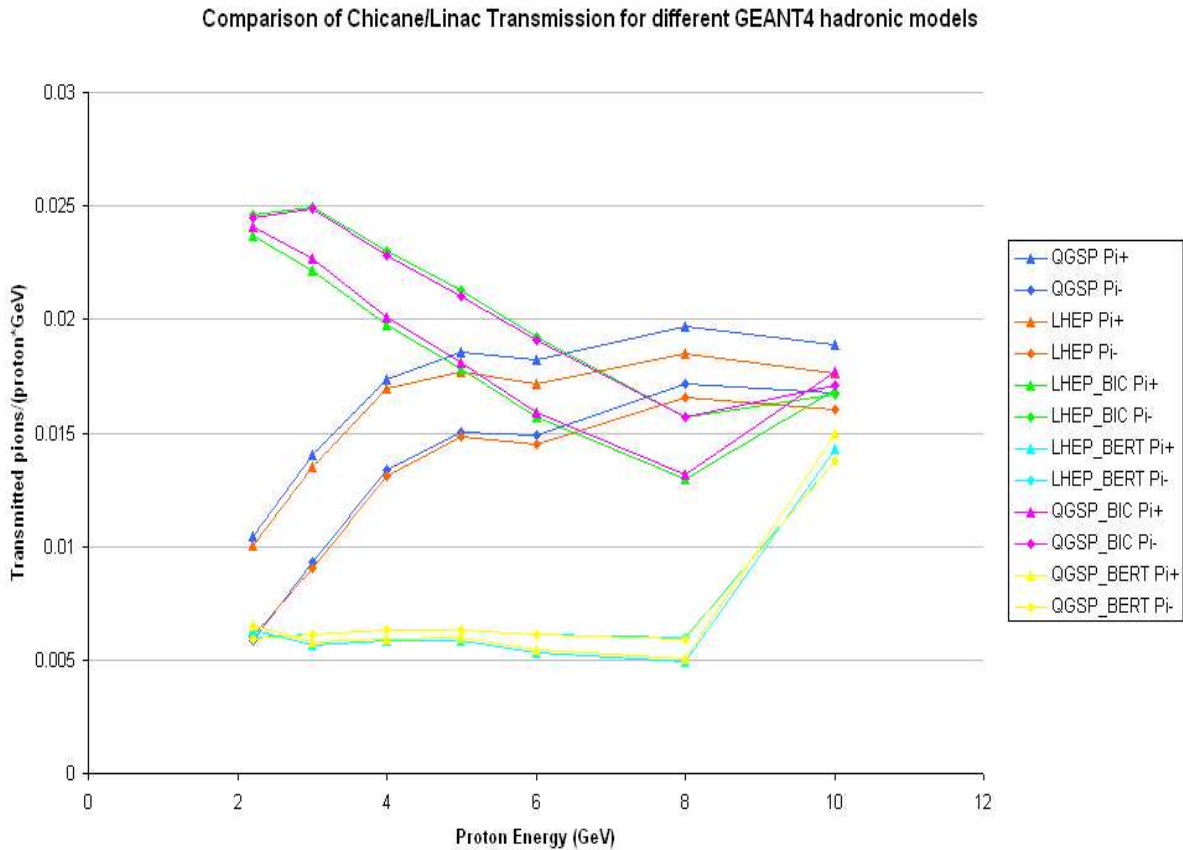


Figure 14: Comparison between GEANT4 hadronic model use-cases for π^+/π^- transmission along 2.2GeV optimised Chicane/Linac Channel

Comparison of Phase Rotator Transmission for different GEANT4 hadronic models

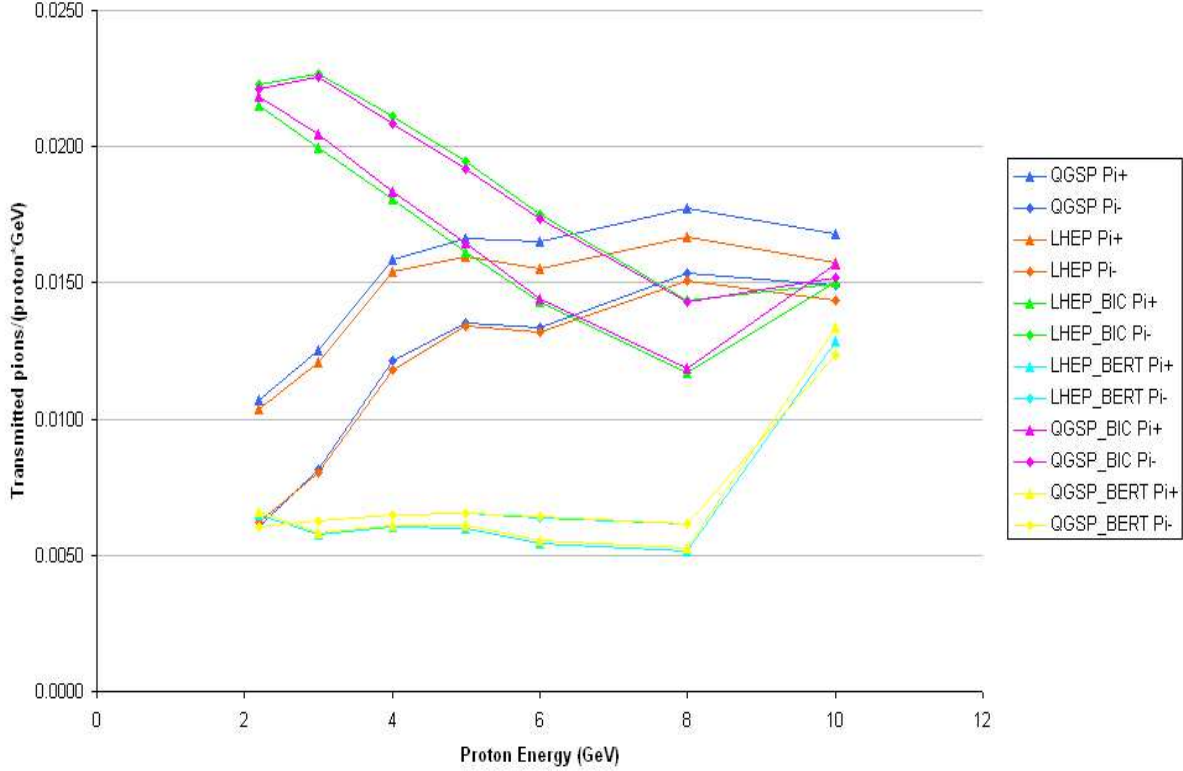


Figure 15: Comparison between GEANT4 hadronic model use-cases for π^+/π^- transmission along 2.2GeV optimised Phase Rotation Channel

7 Conclusion

The results from this paper suggest that the optimal proton energy for a Neutrino Factory derived using GEANT4 and adopting the QGSP hadronic algorithms is 10-15GeV. It can be seen that within the GEANT4 physics use cases there is a large amount of variation in pion yields at low energies from different hadronic models. In terms of comparison between GEANT4 and MARS codes, there are differences in pion yields at all energies with the differences being greatest at the lower end of the range studied. MARS data, as shown in [4] would suggest an optimal energy of around 30GeV for raw yields without probability grids. The largest difference between the two codes is in the calculated angular distributions for outgoing pions. GEANT4 appears to forward focus these pions to a much greater extent than MARS, which in turn leads to much greater yields of capturable pions³⁾. To conclude, these results suggest the optimal proton driver energy for a neutrino factory is in the range 10-15 GeV, while MARS data suggests the optimal energy to be 30GeV. Data from the HARP and other experiments is needed to resolve these discrepancies.

The results obtained by estimating the transmission through both the Chicane/Linac

³⁾Again with caveat on model validity

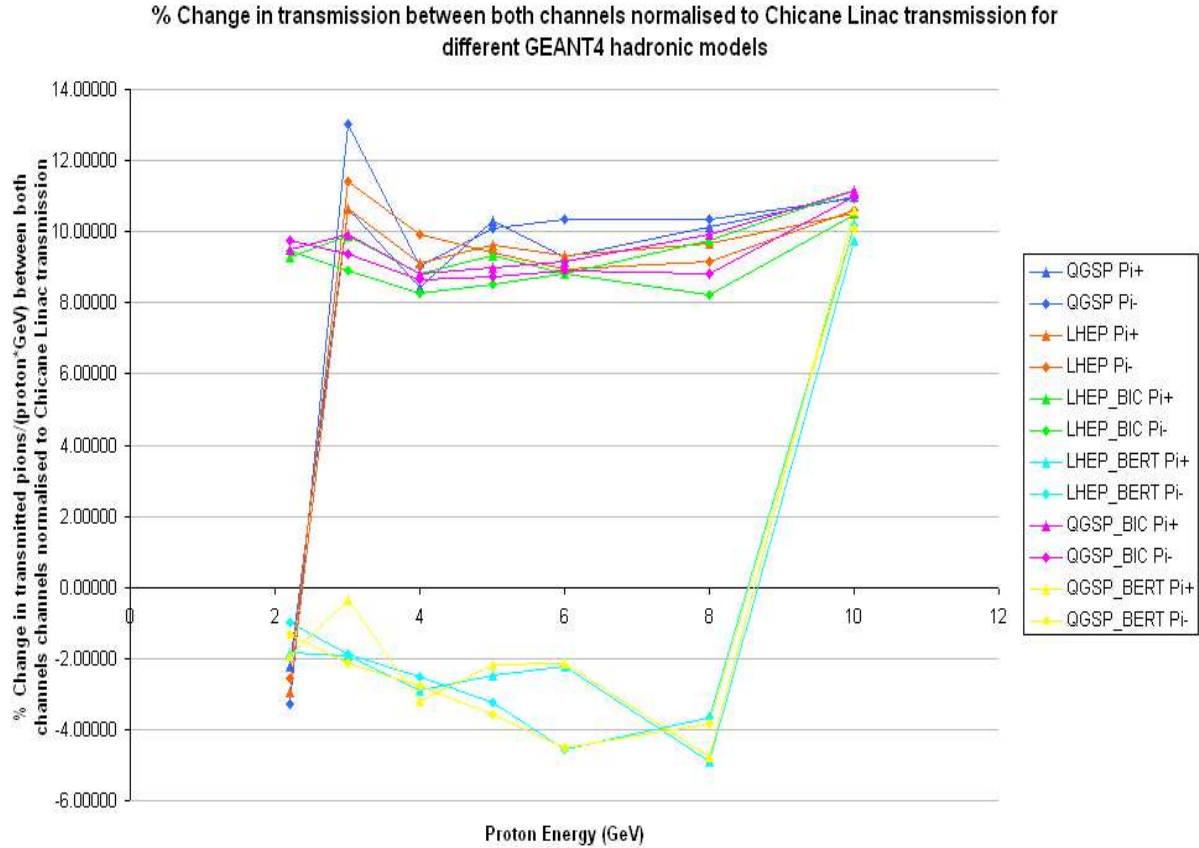


Figure 16: Percentage difference in “beam-power-normalised” transmission between Chicane/Linac and Phase Rotator channels. Percentage normalised to Chicane.Linac transmission

and Phase Rotator front ends show a peak in transmission around 8-10 GeV. It must be noted that the estimation assumes a 2.2GeV optimised channel. Both channels show major differences in transmission for different GEANT4 hadronic physics choices as would be expected from the raw yield studies. It appears that on the whole the Chicane/Linac channel is more efficient at transmitting off-momentum pions. Finally, it appears that for low proton driver energies in both front-ends, the transmission of Bertini Cascade model calculated pions is roughly independent of energy.

8 Further Work

Work to be carried out in the future to build on these results and further investigate different Neutrino Factory target scenarios are:-

1. Run the Probabilitymap program for different magnetic lattice optimisations
2. Compare these results with HARP data when available

References

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Appendix A

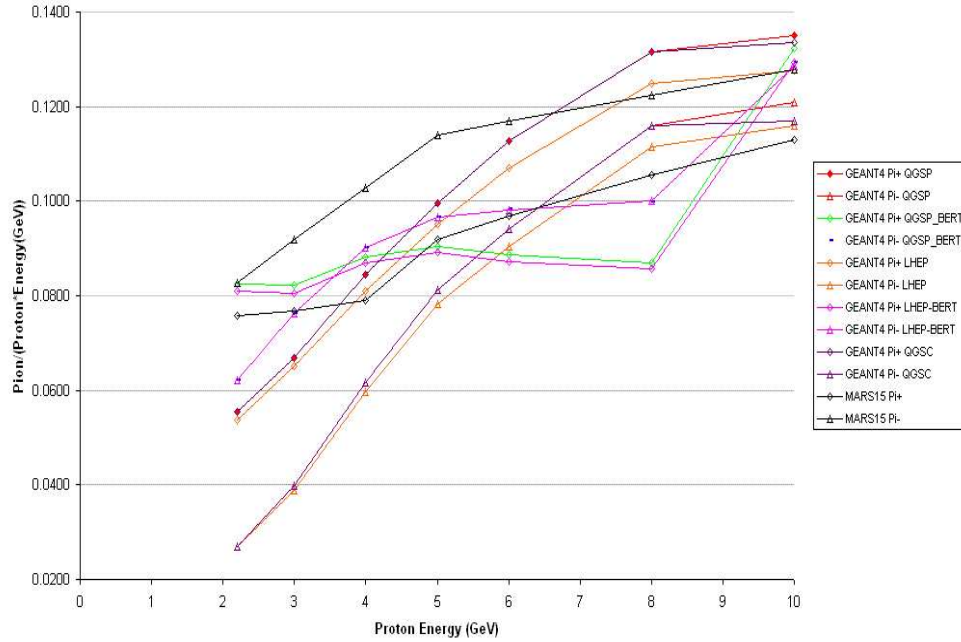


Figure 17: Close-up of Figure 4 highlighting high yield models

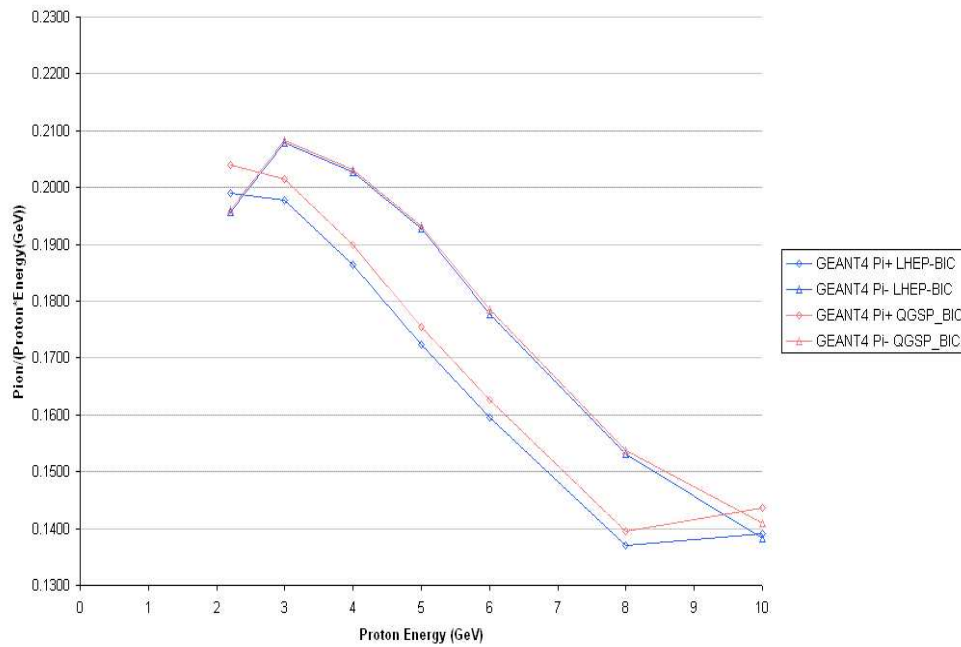


Figure 18: Close-up of Figure 4 highlighting low yield models