QCD Physics - Lecture 1

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Outline

1. Course Introduction
2. QCD Refresher/Introduction
3. *ep* Scattering
4. Quark Parton Model
5. Quantum Chromodynamics
6. Observables

Aim of Course

This course aims to give a survey of experimental measurements of QCD at high and low energies, and introduce techniques used in these measurements

There are many good reference works, the following in particular may be useful:

- Deep Inelastic Scattering - Devenish & Cooper-Sarkar
- QCD and Collider Physics - Ellis, Sterling & Webber
- Quarks and Leptons - Halzen & Martin
- Gauge Theories in Particle Physics - Aitchison & Hey

`http://www-zeus.desy.de/~ferrando/lectures`

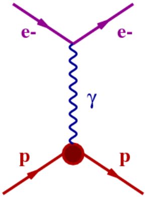
- 4 Interactions: Weak, Electromagnetic, Gravity, Strong
- Strong interaction posited to account for cohesion of nuclei
  - Must be strong (stronger than EM)
  - Must only be effective over short distances
- Original Theory: Yukawa interaction (1935)
  - Model on QED
  - Use a massive $\mathcal{O}(100\ \mathrm{MeV})$gauge boson to account for finite range
  - Ideal candidate π discovered in cosmic ray + accelerator experiments in the 1940s
  - Theorists made renormalisable field theories where nucleons interacted with each other via the exchange of pions, conserving isospin + strangeness
- However perturbation expansions of QFTs of strong interaction broke down

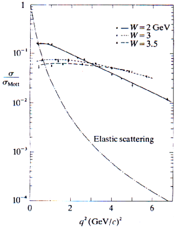
- In QED, succesive complicated diagrams for a process are suppressed by a factor $e^2 = \left(\frac{1}{137}\right)^2$ per loop. Meaning that only the simplest diagrams make an appreciable contribution to the total cross section
- Strong interaction coupling g_S^2 was measured at around 15 for hadrons, so that extra loops make a very large contribution
- Thus perturbative theories broke down, the first term could not be relied upon since its magnitude was so much smaller than subsequent terms
- To make any prediction from theory one must sum the whole series, which is not possible

Aces and Quarks

- In 1964 Gell-Mann and Zweig independently proposed that hadrons such as the proton were composite particles built from three basic blocks now known as up, down and strange
- The proposed particles had very unusual properties:
    - spin $\frac{1}{2}$ and baryon number $\frac{1}{3}$
    - u and d form an isospin doublet with 0 strangeness
    - s an isospin 0 singlet with unit strangeness
    - non-integral charge: $Q_u = +\frac{2}{3}$, $Q_{d,s} = -\frac{1}{3}$
- Mesons formed by $q\bar{q}$, nine combinations available, octet and singlet
- Baryons formed by qqq, in SU(3) only decuplets, octets and singlets available

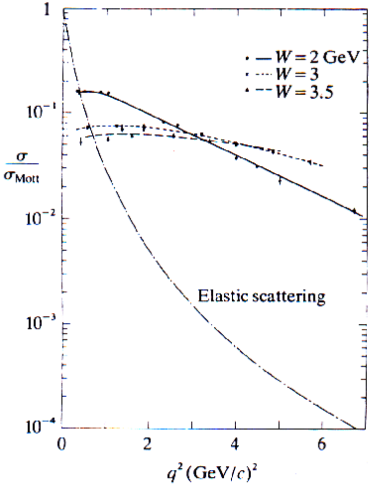
Elastic *ep* Scattering





- In the late 60s, scaling was observed at SLAC
- SLAC used a 22 GeV linear electron accelerator
- elastic *ep* scattering observed
- Elastic *ep* scattering can be understood entirely in terms of *QED* as the exchange by *p* and *e* of a single γ. *e* & q structureless particles whereas *p* an extended object that should show structure
- First measurements showed that σ at large angles in *ep* scattering was much smaller than in *ee* scattering
  - Electrons seemed to act as ‘hard’ objects, bouncing off one another
  - Proton seemed to be a diffuse large object exerting a smaller impulse on passing particles

Elastic *ep* Scattering



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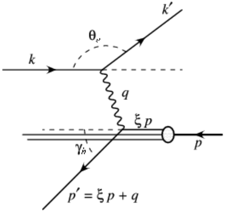
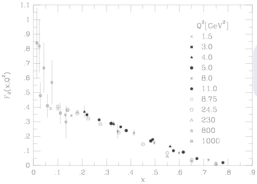
Inelastic *ep* Scattering

- Inelastic *ep* scattering was also thought to be mediated by single photon exchange between proton and electron, and it was thought that the most important process would be resonance production
- Analysis of the data at low electron scattering angle showed the expected behaviour
- At large angles there remained a large measured cross section, despite the disappearance of individual resonance peaks
- The inelastic *ep* cross section was behaving as though the proton contained hard, point-like scattering centres

Inelastic *ep* Scattering





In high energy $ep \rightarrow X$ we can define variables:

$$q = k - k', \;\; Q^2 = -q^2, \;\; M^2 = p^2$$
$$x = \frac{Q^2}{2p.q}, \;\; y = \frac{q.p}{k.p}$$

Structure functions F_1 and F_2 are defined in terms of:

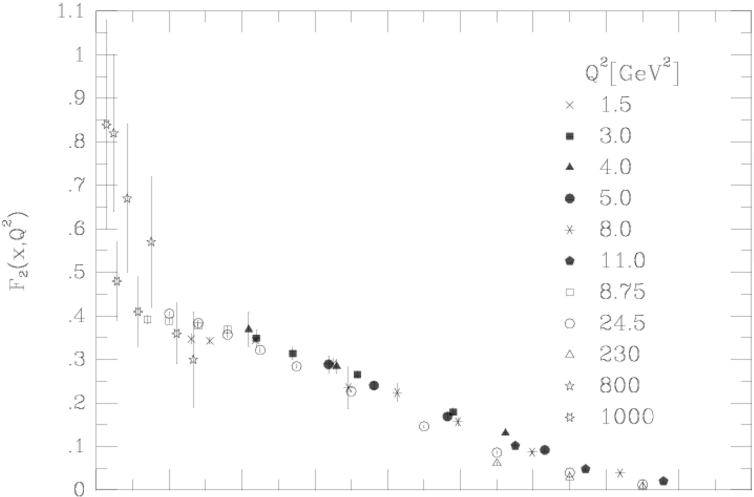
$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\{[1+(1-y)^2]F_1 + \frac{(1-y)}{x}(F_1 - 2xF_2)\}$$

F_1,F_2 are functions of x,Q^2. Björken predicted that for $Q^2 \rightarrow \infty$, and x fixed the F_i depend only on x

$$F_i(x, Q^2) = F_i(x)$$

Björken scaling → γ scatters off (charged) point-like proton constituents.

Inelastic *ep* Scattering



F_1, F

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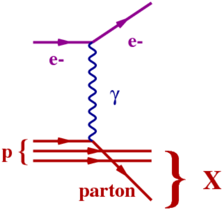
Quark-Parton Model



- QPM developed by Feynman to account for scaling
- Proton constituents are called partons
- Björken scaling $\rightarrow \gamma$ scatters off (charged) point-like proton constituent
- Partons, Quasi-free, regarded as independent over the time of collisions
- Deep Inelastic Scattering (DIS) in the QPM is single γ exchange between the e and a parton
- F_1, F_2 measure the momentum distribution of the charged partons within the proton in terms of x (momentum fraction of proton)
- Quark-Parton Model predicts scaling

- Successes:
  - Provides a method to deal with strong interactions by focusing on their effect on parton distributions
  - With parton distributions, electron-hadron processes became calculable in terms of first order QED processes
- Questions:
  - What are the partons? Are they Quarks?
  - Partons must be Quasi-free to account for scaling, but constituent quark model requires strong inter-quark forces
- Need an extra QPM assumption to stop quarks flying out of the proton after scattering. (No quarks observed in remnants of collisions)
- Feynman showed that scaling would be observed in DIS irrespective of parton nature

- We can express the contribution of each quark to F_i using the fact that the interactions are from QED:

$$F_2(x) = \sum_{q,\bar{q}} e_q^2 xq(x)$$

  $q(x)$ is the probability of finding a parton with momentum fraction between x and $x + \mathrm{d}x$
- Relative magnitudes of F_1 and F_2 are determined by spin.
- $R = F_2 - 2xF_1$ was measured as approximately 0 (Callan-Gross relation), as would be expected in the spin $\frac{1}{2}$ case
- 3 quark (uud) model of protons overestimated the measured size of the structure functions by approximately 2
- Break down quark component into:
  - "Valence" : minimum SU(3) composition of the hadron
  - "Sea" : SU(3) singlet cloud containing indefinite number of quarks

Sum Rules

- Probed at a scale Q^2, sea contains all flavours with $M \ll Q$
- at $Q \approx 1 GeV$ appropriate PDFs are:

$$u(x) = u_V x + u_S(x),\ d(x) = d_V x + d_S(x),\ s(x) = s_S(x)$$

expect that $\bar{u}(x) = \bar{d}(x) = s(x) = \bar{s}(x)$

- Expect the proton momentum sum rules, to hold:

$$\int_0^1 u_V(x)\,\mathrm{d}x = 2 \text{ and } \int_0^1 d_V(x)\,\mathrm{d}x = 1$$

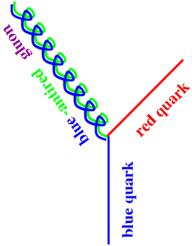
- It was measured that:

$$\sum_q \int_0^1 q(x) + \bar{q}(x) \sim 0.5$$

- 50% of proton momentum is not carried by quarks

- In QFT no realistic interacting theory predicted scaling
- Success of QPM model and scaling in DIS explained with demonstration of Asymptotic Freedom in gauge theories
- Gauge theories are invariant under local transformations of fields, thus predictions are unchanged by such transformations
- Asymptotic Freedom means that at high momenta gauge theories behave as free, non-interacting field theories
- important steps:
  - 1972: Gross & Coleman - no renormalisable field theory except gauge theories, could account for scaling
  - 1973: Politzer & Wilczek - asymptotic freedom of gauge theories
  - 1973: Gross & Coleman - no theory without gauge fields asymptotically free
- A field theory of the strong interaction must be a gauge theory

A Gauge Theory of the Strong Interaction



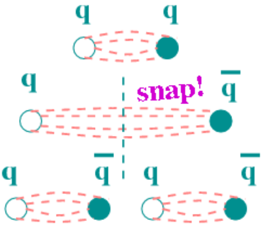
- Assume the fundamental gauge fields are quarks interacting via exchange of gauge vector fields (gluons)
- Since quarks are fermions, total wave function must be antisymmetric under quark-interchange. Wave functions of quarks were represented by three components

$$\Psi_q = \psi_{SU(3)_F}\phi_{\mathrm{spin}}\psi_{\mathrm{space}}$$

  Which is symmetric overall under exchange of quarks.
- Solution : Introduce a new Quantum number → Colour
- Gauge theory of SI describing interactions between quarks and gluons within SU(3) group of colour is known as Quantum Chromo Dynamics (QCD)
- 3 Colour charges, conserved at each vertex, used:
  - quarks are singly charged.
  - gluons doubly charged.

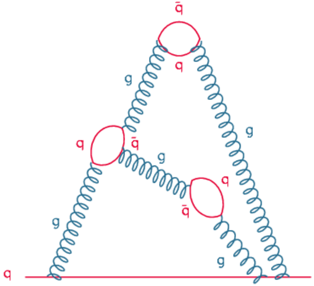
Confinement

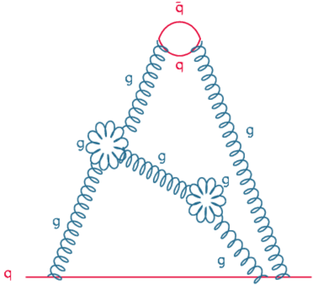
- QCD exhibits the two most important properties of quark and gluon interactions: aymptotic freedom and confinement
- Confinement offers a solution to the problem of the lack of observation of free quarks
- Since gluons also carry colour charge, they can couple directly to other gluons
- The potential energy required to separate quarks increase linearly with their distance
- Quarks and gluons are only seen in colourless hadrons
- Large amounts of energy to break up hadrons
- This energy is needed to create quark-anti-quark pairs



Asymptotic Freedom I

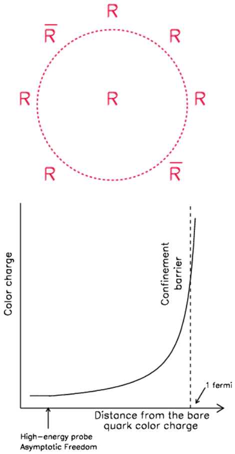
- In QFT a quark can emit a gluon, the gluon can decay to a $q\bar{q}$ pair and so on





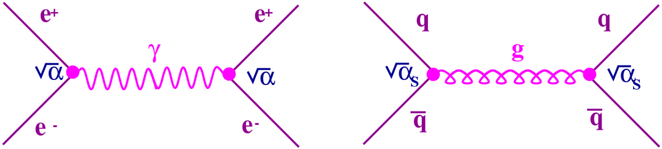
- The $\bar{q}q$ pairs surround the quark and diffuse the effective colour of the quark

Asymptotic Freedom II



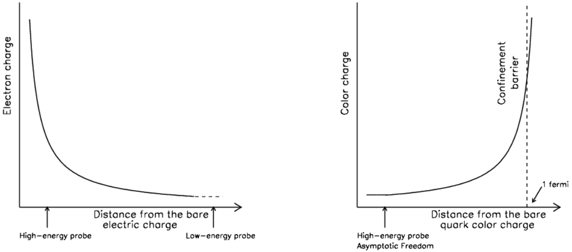
- A red charge for example is surrounded by a cloud of colour dipoles antiscreening the colour charge
- To determine the charge, measure the force experienced by a test charge
- moving the test charge towards the quark leads to decrease in measured charge!
- This antiscreening results in asymptotic freedom
- When very close together, the quarks interact through colour fields of reduced strength, and behave as free non-interacting particles

The Coupling α_S



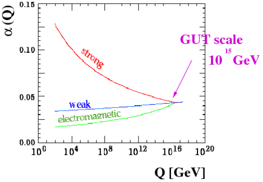
- α is a measure of the strength of the EM interaction.$\alpha = \frac{e^2}{4\pi}$
- In QED, α gives the probability of emitting a photon
- a factor $\sqrt{\alpha}$ is associated with each absorption or emission of a photon by a charge e
- α_S is the QCD analogue to α: $\alpha_S = \frac{g_S^2}{4\pi}$ where g_S is the colour charge
- In QCD, α_S gives the probability of emitting a gluon.
- a factor $\sqrt{\alpha_S}$ is associated with each absorption or emission of a gluon by a colour charge g_S

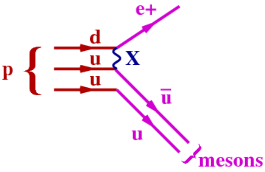
Running Couplings



- In QED the charge seen depends on distance from the particle, due to screening. α depends on the scale, and is said to run
- In QCD the magnitude of the strength also depends on the scale at which we view the particle. So α_S also runs
- In QCD the value of α_S is small at short distances, this is the scale at which perturbation theory works

Grand Unification





- We expect all couplings to merge at a very large (10^{15} GeV) energy scale
- At this stage a Grand Unification theory (GUT) would unify the strong, weak and EM interactions, making them all aspects of the same interaction
- Several models have been proposed, none are entirely satisfactory, so far
- GUT's predict lifetime of 10^{31} years for the proton
- Hitherto there has been no detection of proton decay

Predictions of QCD

To test the validity of any theory, it's predictions must confront experimental data. Important predictions:

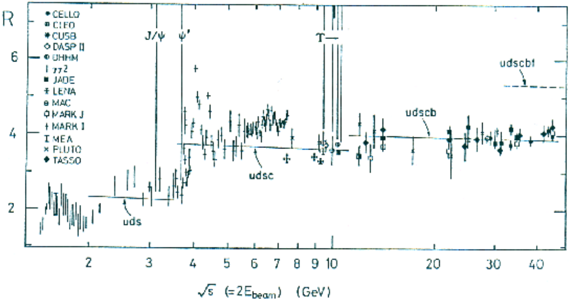
- 3 Colour States per Quark: Cross sections should yield evidence that there are 3 colour degrees of freedom for each quark in a final state
- Scaling Violations: With increasing Q^2, $F_2(x, Q^2)$ should increase at small x and decrease at large x
- Running of α_S: α_S should decrease with increasing Q^2
- Hadronic Jets: Since quarks and gluons cannot be seen in isolation, we should only directly observe hadronic jets
- Gluon Jets: Some jets should originate from gluons
- Gluon Self Coupling: Evidence for a ggg vertex should be observed

Measuring Quark Colour Species I

$$\sigma(e^+e^- \longrightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}, \; \sigma(e^+e^- \longrightarrow q\bar{q}) = n_c \sum_q e_q^2 \frac{4\pi\alpha^2}{3s}$$

- Expect $R = \frac{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}{\sigma(e^+e^- \rightarrow q\bar{q})} = n_c \sum_q e_q^2$
- Well measured μ cross section makes it possible to confirm number, colour states and electromagnetic charge of quarks

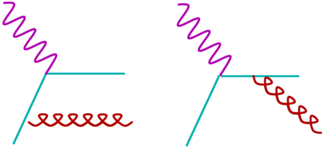
Measuring Quark Colour Species II



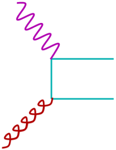
R$=2(u,d,s)$, $10/3(u,d,s,c)$, $11/3(u,d,s,c,b)$, $15.3(u,d,s,c,b,t) \longrightarrow n_c = 3$

Scaling Violation I

- The QPM predicts exact scaling
- QCD predicts that exact scaling is broken by:



Gluon emission by a quark (QCD compton process)



DIS with initial gluons (Boson-Gluon Fusion Process).

- These give rise to terms $\propto \alpha_S \ln Q^2$ which break scaling
- Such logarithmic scaling violations are a property of gauge theories with point like fermion-vector boson couplings

Scaling Violation II

QCD gives the quark parton distribution in the proton to $\mathcal{O}(\alpha_S)$ as:

$$q(x,Q^2) = q_0(x) + \frac{\alpha_S}{2\pi}\left\{q_0(x) \otimes P_{qq}(x) \ln\frac{Q^2}{\kappa^2} + g_0(x) \otimes P_{qg}(x) \ln\frac{Q^2}{\kappa^2}\right\}$$

where P_{ij} are the splitting functions

The Q^2 evolution of $q(x,Q^2)$ is governed by ...

The DGLAP evolution equations

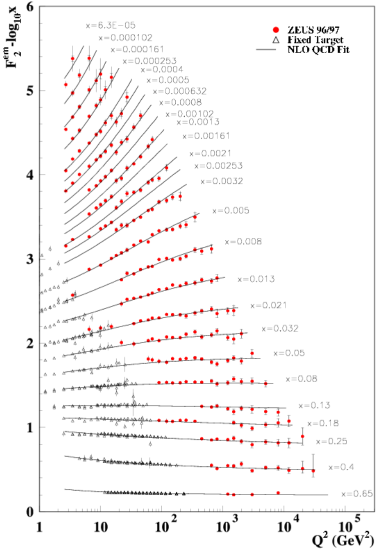
$$\frac{\partial q_{NS}}{\partial \ln Q^2} = \frac{\alpha_S(Q^2)}{2\pi} P_{qq}^{NS} \otimes q_{NS},$$

$$\frac{\partial}{\partial \ln Q^2}\begin{pmatrix} q_S(x,Q^2) \\ g(x,Q^2) \end{pmatrix} = \frac{\alpha_S(Q^2)}{2\pi}\begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q_S(x,Q^2) \\ g(x,Q^2) \end{pmatrix},$$

$$q_S = q + \bar{q},\ q_{NS} = q - \bar{q}$$

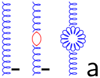
Solutions mean that as Q^2 increases the NS distribution function decreases at large x and increases at small x, so that the phase space for gluon emission by quarks increases as Q^2 increases

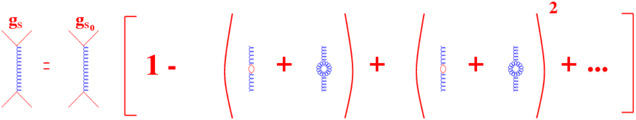
Evidence of Scaling Violation



- Plot showing measurements of $F_2(x, Q^2)$ in DIS from collider (ZEUS) and fixed target (E666,NMC,BCDMS) experiments
- Here we see that F_2 is increasing with Q^2 for small x and decreasing at large x
- Note the large increase in Q^2 and x ranges due to the HERA data

Running of α_S I

- The magnitude of the colour charge is modified by antiscreening
- The gluon propagator  is modified to  at leading order
- Summing to all orders gives:

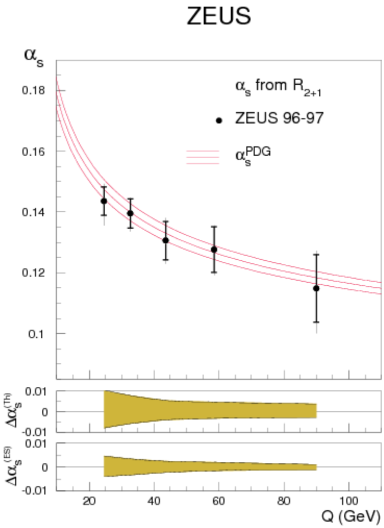


- The sum of a geometric series giving $g_S^2 = g_{S_0}^2 \left(\frac{1}{1+I(Q^2)}\right)$
- We expect that α_S will run with Q^2

Running of α_S II

- In QCD, $I(Q^2) = \frac{\alpha_S(\mu^2)}{12\pi}(33 - 2N_F)\ln\frac{Q^2}{\mu^2}$, μ, a reference scale
- At sufficiently low Q^2, the effective coupling becomes large
  The scale at which this happens is
  $\Lambda^2 = \mu^2 \exp\left[\frac{-12\pi}{(33-2N_F)\alpha_S(\mu^2)}\right]$
- Then: $\alpha_S(Q^2) = \frac{12\pi}{(33-2N_F)\ln\frac{Q^2}{\mu^2}}$
- $\alpha_S(Q^2)$ decreases with increasing Q^2 and is small for short distance interactions
- for $Q^2 \gg \Lambda^2$, perturbative description in terms of quarks and gluons interacting weakly is possible
- for $Q^2 \approx \Lambda^2$, the quarks and gluons are tightly bound in hadronic states and perturbation theory is inapplicable
- The important parameter Λ is not predicted by theory and must be determined by experiment. Recent measurements give $\Lambda = 208^{+25}_{-23}$ MeV

Running of α_S III



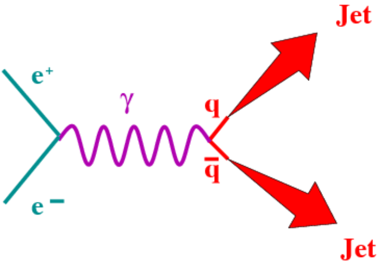
The fraction of DIS events containing 2 jets relative to the fraction containing 1 jet is proportional to α_S

α_S from jet rates in DIS at HERA

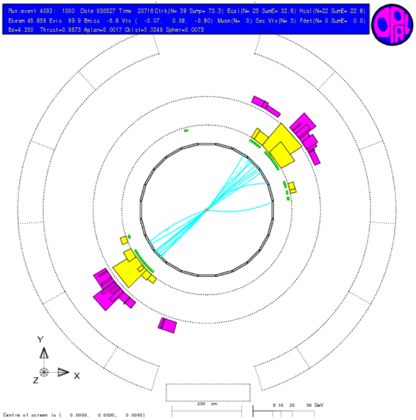
$$\alpha_S(M_Z) = 0.1166 \pm 0.0019(\mathrm{Stat}) \;^{+0.0024}_{-0.0033}(\mathrm{Exp}) \;^{+0.0057}_{-0.0044}(\mathrm{Theo})$$

Jets

- Quarks and Gluons cannot be observed directly
- The collision of very high energy electron + positron beams offers an opportunity to look for direct evidence for quarks
- Photons are produced in e^+e^- collisions
- Can produce $q\bar{q}$
- $q\bar{q}$ pull apart producing 2 jets travelling in the same direction as the quarks and approximately opposite to each other

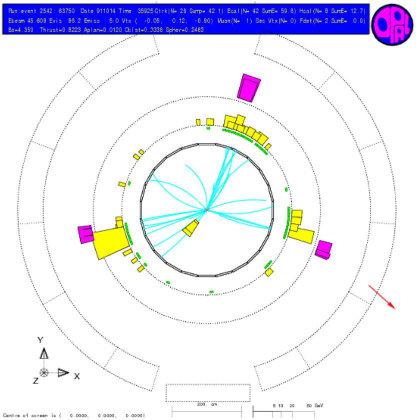


Evidence for Jets



- 1^{st} observation of back-to-back dijet events in $e^+e^- \rightarrow q\bar{q}$ at SPEAR 1975
- The collision of very high energy $e^- + e^+$ beams offers an opportunity to look for direct evidence for quarks
- Back to back collimated bunches of tracks from charged hadrons seen in the central tracking detector
- Back to back hadronic clusters in the calorimeter

Gluons & 3-Jet Events



- $\mathcal{O}(\alpha_S)$ correction to $e^+e^- \longrightarrow q\bar{q}$ gives events with 3 jets in the final state
- Jets are coplanar to conserve momentum
- first direct evidence for gluons by observation of 3 jet events at PETRA in 1979

Gluons & 3-Jet Events

- The leading order cross section: $\sigma_0(e^+e^- \rightarrow q\bar{q}) = 3e_q^2\frac{4\pi\alpha^2}{3s}$, where s is the COM energy squared, is modified at $\mathcal{O}(\alpha_S)$ by gluon emission diagrams
- the modified cross section is:

$$\sigma(e^+e^- \rightarrow q\bar{q}) + \sigma(e^+e^- \rightarrow q\bar{q}g) = \sigma_0(1 + \frac{\alpha_S(s)}{\pi})$$

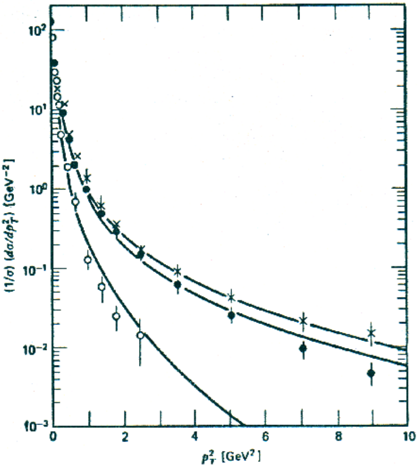
- This can be written in terms of transverse momentum (p_T) between the q and $\bar{q}$

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}p_T} \sim \alpha_S\frac{1}{p_T^2}\ln\left(\frac{s}{4p_T^2}\right)$$

- For fixed p_T the cross section increases with increasing s

Gluons & 3-Jet Events

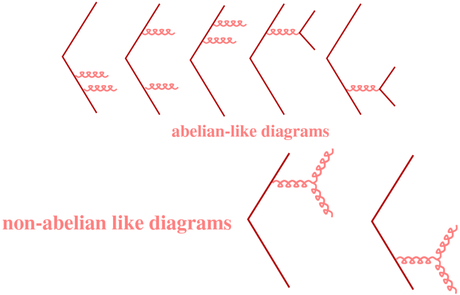
- p_T is non-zero only when there is gluon-emission.
- The hadrons from one of the quarks should also have large p_T



- Measurement of the p_T distribution w.r.t. the thrust axis of hadrons at different s at PETRA
- For fixed p_T the distribution increases with increasing s
- For s the distribution falls with increasing p_T

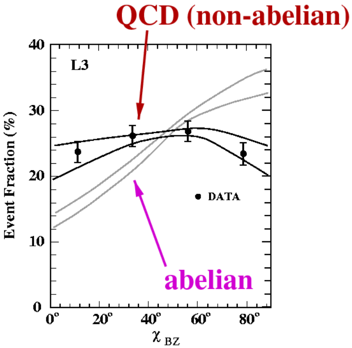
Triple Gluon Vertex I

- QCD is based on the non-abelian SU(3) group which introduces self-coupling of the bosons. QED is based on the abelian group U(1)
- Due to the gluon self-coupling, the *ggg* vertex should be observed in events containing at least 4 jets in the final state
- Diagrams contributing to σ for four jet events:



Triple Gluon Vertex II

- 4-jet events have been observed at LEP
- Variables were devised to highlight the non-abelian nature of QCD in contrast to abelian theories such as $[U(1)]^3$
- Bengtsson-Zerwas angle χ_{BZ} the angle between the planes defined by the two lowest and two highest energy jets
- Nachtmann-Reiter angle θ^*_{NR}, angle between the momentum vector of the differences of jets 1,2 and jets 3,4
- Angle between two lowest energy jets
- These have been measured at LEP
- Data clearly favours non-abelian theory



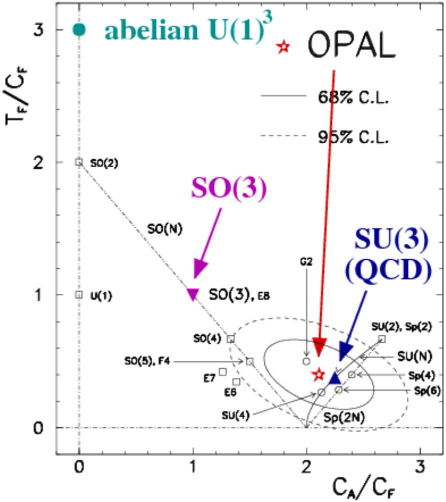
Colour Factors I

- Dynamics of a gauge theory are defined by commutation relations between its generators T^i, $[T^i, T^j] = i \sum_k f^{ijk} T^k$
- In perturbative calculations the average and sum over all possible colour configurations in the initial and final states lead to the combinatoric factors C_A, C_F and T_F.

$$\sum_{k,\eta} T^k_{\alpha\eta} T^k_{\eta\beta} = \delta_{\alpha\beta} C_F, \sum_{\alpha,\beta} T^m_{\alpha\beta} T^n_{\beta\alpha} = \delta^{mn} T_F, \sum_{j,k} f^{jkm} f^{jkn} = \delta^{mn} C_A$$

- C_A, C_F and T_F are known as the colour factors and are the physical manifestations of the underlying group structure
- in QCD they represent the relative strength of the processes $q \to qg$, $g \to gg$ and $g \to q\bar{q}$
- Simulataneous measurement of C_A/C_F and T_F/C_F at in e^+e^- collisions has been made

Colour Factors II



- Predictions:

Group	C_A/C_F	T_F/C_F
$SU(3)$	$9/4$	$3/8$
$[U(1)]^3$	0	3
$SO(3)$	1	1

- Measured values are:

$$C_A/C_F = 2.11 \pm 0.16(\text{St.}) \pm 0.28\ (\text{Sy.})$$

$$T_F/C_F = 0.40 \pm 0.11(\text{St.}) \pm 0.14\ (\text{St.})$$

- SU(3) is clearly favoured

Summary/Conclusions

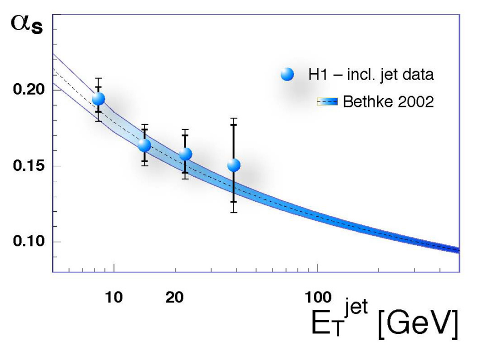
- QCD is a well established theory of the strong interaction
- It explained many known features of the strong interaction
- Many predictions were made and have been confirmed by experiment

And...

Nobel Prize

Nobel Prize in Physics 2004: Gross, Politzer, Wilczek

"For the discovery of asymptotic freedom in the theory of the strong interaction"



- HERA + Experiments
- Deep Inelastic Scattering
- Proton Structure Functions
- Charged Current DIS
- Parton Distribution Functions

- QCD in e^+e^- annihilation
- Jets in e^+e^- annihilation
- Event Shapes
- Triple Gluon vertex
- QCD at Future e^+e^- colliders
- Photon Structure

- Measuring α_S
- Jets
- Jet Structure
- NLO fits
- Other Measurements of α_S

- QCD in $p\bar{p}$
- Jet Production
- Drell-Yan Scattering
- Direct/Prompt Photons
- Heavy Quark Production

- Recent Workshops
- PDFs and the LHC
- Energy Flow
- Heavy Quark Production revisited