QCD Physics - Lecture 2

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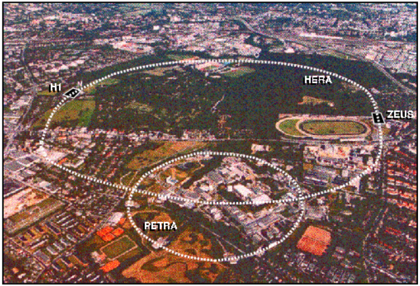
Outline

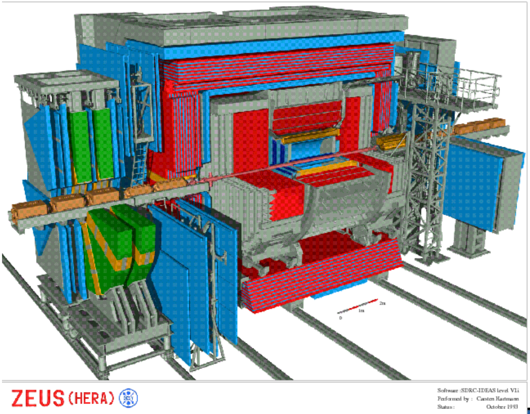
The HERA Facility



- HERA is the only *ep* collider in existence
- Collides 920 GeV p and 27.5 GeV $e^{\pm}$ at H1 & ZEUS
- $\sqrt{s}$ order of magnitude more than past DIS experiments
- Allows probing of very high Q^2 and low x regions of DIS
- HERMES -fixed target exp. studying nucleon spin structure

ZEUS & H1

- Taking data since 1992
- ZEUS optimised for precision measurements of hadronic final state
- H1 optimised for precision measurements of the scattered lepton
- HERA experiments have published on a wide variety of topics
- 2003-2007 is HERA II running with luminosity upgrade + polarised leptons



A rich variety of physics topics is available for Study at HERA:

High Q^2

- Structure of Proton
- EW physics: $\sigma_{\mathrm{NC,CC}}$ DIS
- Rare Standard Model processes
- Physics beyond the SM

QCD/Hadronic Final State

- Photon structure.
- Jet production.
- Particle production.
- Measurements of α_{S}.

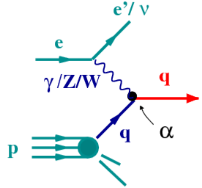
Heavy Flavour

- Production of c, b quarks
- Hadronisation of heavy quarks
- $F_2^{c\bar{c}}$,$F_2^{b\bar{b}}$

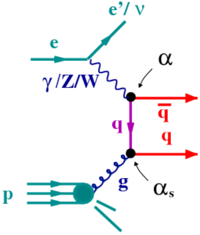
Diffractive/Low x

- Study of events with a large rapidity gap.
- Vector Meson production.

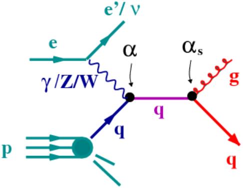
- DIS of leptons on nucleons has been an important tool for understanding nucleon structure and many elements of the SM
- At HERA DIS processes are studied at $\sqrt{s} \approx 320$ GeV and $Q^2 > M_W^2, M_Z^2$
- Unique tests of the SM and it's extensions are possible in this regime
- Neutral and charged current interactions up to $\alpha\alpha_S$:



Quark-Parton Model

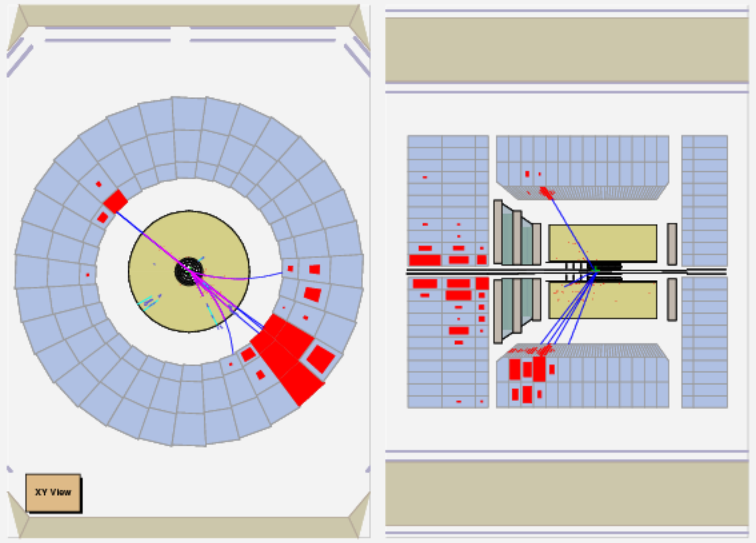


Boson-Gluon Fusion

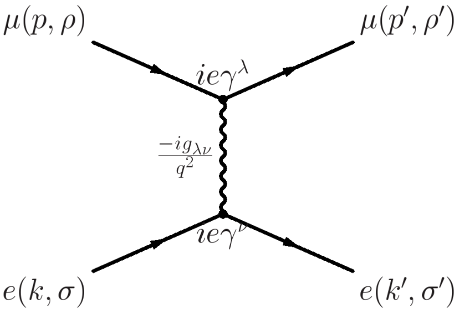


QCD Compton

NC DIS Event



Calculating σ_{DIS} I



consider elastic $e\mu$ scattering

lepton currents

$$j_e^\nu = ie\bar{u}(k',\sigma')\gamma^\nu u(k,\sigma) \quad (1)$$

$$j_\mu^\lambda = ie\bar{u}(p',\rho')\gamma^\lambda u(p',\rho') \quad (2)$$

propagator

$$\frac{-ig_{\lambda\nu}}{q^2} \quad (3)$$

$$\mathcal{M} = i\frac{ee'}{q^2}[\bar{u}(k',\sigma')\gamma_\lambda u(k,\sigma)][\bar{u}(p',\rho')\lambda^\mu u(p,\rho)]$$

For unpolarised σ, the initial spin states must be averaged over.

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{e^2e'^2}{q^4}L_e^{\lambda\nu}L^\mu_{\lambda\nu}$$

Where: $L_e^{\lambda\nu} = 2(k'^\lambda k^\nu + k'^\nu k^\lambda - (k'.k)g^{\lambda\nu})$

Calculating σ_{DIS} II

Contract the leptonic tensors

$$L_e^{\lambda\nu} = 2(k'^{\lambda}k^{\nu} + k'^{\nu}k^{\lambda} - (k'.k)g^{\lambda\nu})$$
$$L^{\mu}_{\lambda\nu} = 2(p'_{\lambda}p_{\nu} + p'^{\nu}p_{\lambda} - (p'.p)g_{\lambda\nu})$$
$$L_e.L^{\mu} = 8[(k'.p')(k.p) + (k'.p)(k'.k)]$$

Rewrite in terms of the Mandelstam variables

$$s = (k+p)^2 = (k'+p')^2, t = (k-k')^2 = (p'-p)^2, u = (k-p')^2 = (k'-p)^2$$
$$L_e.L^{\mu} = 2(s^2 + u^2).$$

substitute $y = \frac{(p.q)}{(p.k)} = \frac{u}{s} + 1$

$$\frac{1}{4}\sum_{\mathrm{spins}} |\mathcal{M}|^2 = \frac{e^2 e'^2}{Q^4} 2s^2[1 + (1-y)^2]$$

Insert phase space and flux factor

$$\frac{\mathrm{d}\sigma}{\mathrm{dy}} = \frac{e^2 e'^2}{8\pi Q^4}[1 + (1-y)^2]s \longrightarrow \frac{\mathrm{d}\sigma}{\mathrm{dy}} = \frac{2\pi\alpha^2}{Q^4}[1 + (1-y)^2]s$$

One isotropic contribution from same handed spin directions

Calculating σ_{DIS} III

Calculation for $e\mu$ scattering applies to eq scattering. However we change the variables: the q contains a fraction x' of the proton momentum. meaning $p \rightarrow x'p$ gives $s \rightarrow x's$ so that:

$$\frac{\mathrm{d}\sigma}{\mathrm{dy}} = \frac{2\pi\alpha^2}{Q^4}[1 + (1 - y)^2]x'se_i^2$$

where e_i is the charge of the quark In the QPM we can interpret lh scattering as the incoherent sum of l-parton scattering. We write:

$$\frac{\mathrm{d}\sigma}{\mathrm{dxdy}} = \frac{2\pi\alpha^2}{Q^4}[1 + (1 - y)^2]s\sum_i x'e_i^2 q_i(x)$$

$(q(x_i)$: probability quark q_i carries a fraction) x of the hadron momentum)

The distribution $xq_i(x)$ is a parton distribution function (PDF)

Rewrite the double differential cross section using $Q^2 = sxy$

$$\frac{\mathrm{d}\sigma}{\mathrm{dxdQ^2}} = \frac{2\pi\alpha^2}{xQ^4}[1 + (1 - y)^2]\sum_i x'e_i^2 q_i(x)$$

Calculating $\sigma_{\rm DIS}$ IV

Compare QPM result to general formula for *lh* scattering.

$$d\sigma \sim L^{e}_{\mu\nu} W^{\mu\nu}$$

where $W^{\mu\nu}$ is the hadronic tensor analogous to the lepton tensor.

General form for $W^{\mu\nu}$

$$\begin{aligned} W^{\mu\nu} &= -W_1 g^{\mu\nu} + \frac{W_2}{m^2} p^\mu p^\nu - i\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta \frac{W_3}{2m^2} + \frac{W_4}{m^2} q^\mu q^\nu \\ &\quad + \frac{W_5}{m^2}(p^\mu q^\nu + p^\nu q^\mu) + i(p^\mu q^\nu - p^\nu q^\mu)\frac{W_6}{2m^2} \end{aligned}$$

$\epsilon^{\mu\nu\alpha\beta}$ is the totally antisymmetric rank 2 tensor which is $+1$(-1) when $\mu\nu\alpha\beta$ is an even (odd) permutation of 0123 and 0 otherwise.

W_6 term disappears for unpolarised scattering since $L^{\mu\nu}$ is symmetric. For γ scattering the parity violating W_3 term is also discarded.

Calculating σ_{DIS} V

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{m^2} p^\mu p^\nu + \frac{W_4}{m^2} q^\mu q^\nu + \frac{W_5}{m^2}(p^\mu q^\nu + p^\nu q^\mu)$$

Simplify $W^{\mu\nu}$ using conservation of current at the hadronic vertex

$$q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$$

giving:

$$W_5 = -\frac{p.q}{q^2} W_2$$

and:

$$W_4 = \left(\frac{p.q}{q^2}\right)^2 W_2 + \frac{M^2}{q^2} W_1$$

So that:

$$W^{\mu\nu} = W_1\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + W_2 \frac{1}{M^2}\left(p^\mu - \frac{p.q}{q^2} q^\mu\right)\left(p^\nu - \frac{p.q}{q^2} q^\nu\right)$$

Calculating σ_{DIS} VI

Following the calculation with this hadronic tensor (see e;g; Halzen and Martin) gives:

$$\frac{\mathrm{d}^2\sigma^{e^\pm p}}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{xQ^4}\left[y^2 x F_1(x,Q^2) + (1-y)F_2(x,Q^2)\right]$$

where

$F_1(x,Q^2) = MW_1(\nu,Q^2)$, $\nu = (p.q)$

and

$F_2(x,Q^2) = \nu W_2(\nu,Q^2) = \frac{p.q}{M} W_2(x,Q^2)$.

define:

$F_L = F_2 - 2xF_1$

then:

$$\frac{\mathrm{d}^2\sigma^{e^\pm p}}{\mathrm{d}x\mathrm{d}Q^2} = \frac{2\pi\alpha^2}{xQ^4}\left[[1+(1-y)^2]F_2(x,Q^2) - y^2 F_L(x,Q^2)\right]$$

QPM Predictions

Compare our general result to the quark parton model result:

$$\frac{\mathrm{d}^2\sigma^{e^{\pm}p}}{\mathrm{d}x\mathrm{d}Q^2} = \frac{2\pi\alpha^2}{xQ^4}\left[[1+(1-y)^2]F_2(x,Q^2) - y^2F_L(x,Q^2)\right]$$

$$\frac{\mathrm{d}\sigma}{\mathrm{dxdQ^2}} = \frac{2\pi\alpha^2}{xQ^4}[1+(1-y)^2]\sum_i x' e_i^2 q_i(x)$$

This implies that $F_2(x,Q^2) = \sum_i x' e_i^2 q_i(x)$ SCALING!

Another prediction is the Callan-Gross relationship $F_L = 0$:

$$2xF_1(x) = F_2(x)$$

This is a consequence of the partons having spin $1/2$.

Full cross section includes Parity Violating term, neglected in this calculation.

Double differential cross section for inclusive *ep* scattering:

$$\frac{\mathrm{d}^2\sigma^{e^{\pm}p}}{\mathrm{d}x\mathrm{d}Q^2} = \frac{2\pi\alpha^2}{xQ^4}\left[\left[Y_+F_2(x,Q^2) \mp Y_-xF_3(x,Q^2) - y^2F_L(x,Q^2)\right](1+\delta_r(x,Q^2)\right)$$

$Y_{\pm} = 1 \pm (1-y)^2$

F_L is the Longitudinal Structure Function.

xF_3 is the parity violating term.

δ_r is the electroweak radiative correction.

Structure function F_2 contains contributions from virtual photon and Z_0 exchange:

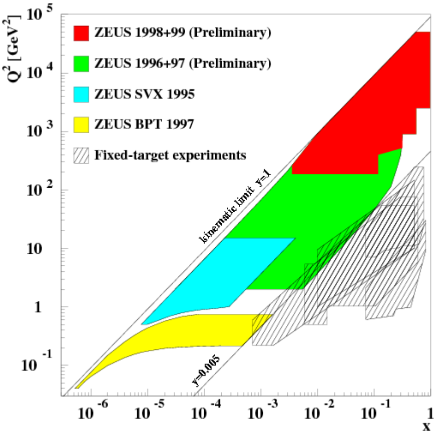
$$F_2 = F_2^{\mathrm{em}} + \frac{Q^2}{(Q^2+M_Z^2)}F_2^{\mathrm{int}} + \frac{Q^4}{(Q^2+M_Z^2)}F_2^{\mathrm{wk}} = F_2^{\mathrm{em}}(1+\Delta F_2)$$

F_2^{em} is the contribution from the photon.

F_2^{wk} is the contribution from the Z_0.

F_2^{int} is the interference term.

Kinematic Range of Measurements



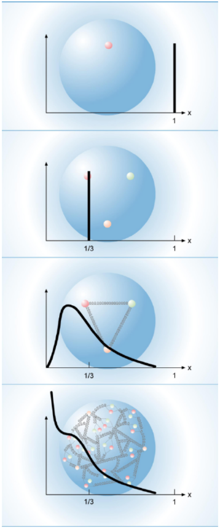
- HERA: ZEUS, NC cross sections:

  $x > 10^{-6}$ and

  $0.05 < Q^2 < 10^5\ \mathrm{GeV}^2$
- Fixed Target
  - μ-induced F_2 from BCDMS, NMC, E665.
  - Deuterium-target data from NMC and E665.
  - NMC data on F_D^2/F_2^p
  - CCFR xF_3 data.

  $x > 6.10^{-4}$ and

  $0.2 < Q^2 < 200\ \mathrm{GeV}^2$

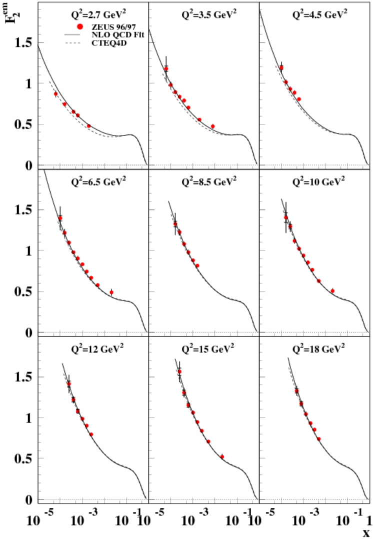


A single particle

Three valence quarks

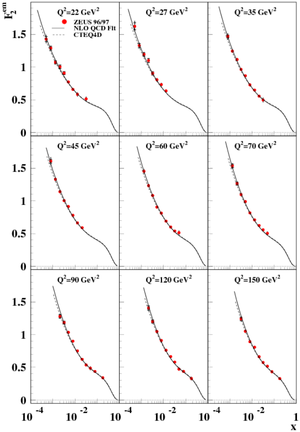
Three valence quarks with interactions

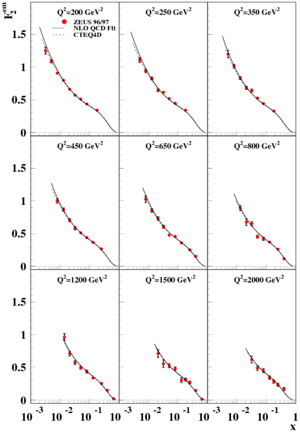
Valence and sea quarks with interactions



F_2

Measurement of F_2 for $2.7 < Q^2 < 6 \times 10^5\ \mathrm{GeV}^2$ has statistical & systematic uncertainties below 2% in most of the (x, Q^2) region

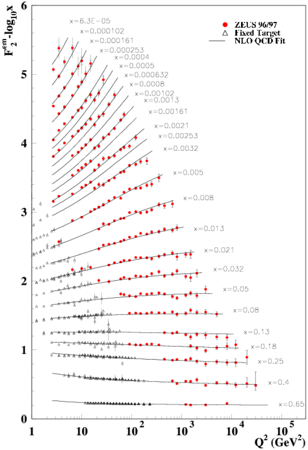


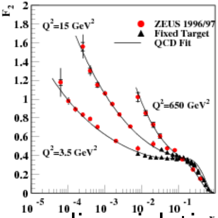


Good agreement with fixed target experiments at large x values

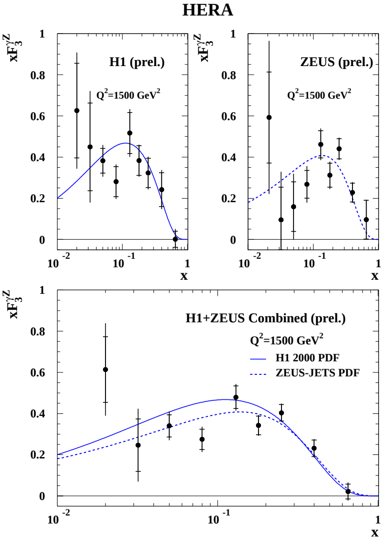
F_2

Measurement of F_2 for $2.7 < Q^2 < 30000\mathrm{GeV}^2$ and 6×10^5.





- Strong scaling violations for $x < 0.02$
- Measured $x - Q^2$ behaviour can be described by DGLAP equations over the whole kinematic range



- Parity violating part of Z-exchange (xF_3) in NC DIS makes a -ve contribution to $\sigma(e^+p)$ and a +ve contribution to $\sigma(e^-p)$
- Can be measured by subtracting e^+p from e^-p cross section
- SM expectation describes data well.

How do we measure the structure functions?

- Define a cross section ($\frac{\mathrm{d}\sigma_{\mathrm{pred}}}{\mathrm{d}Q^2\mathrm{d}x}$) dependent on $F_2(x, Q^2)$
- Measure $\frac{\mathrm{d}\sigma_{\mathrm{pred}}}{\mathrm{d}Q^2\mathrm{d}x}$
- Extract $F_2(x, Q^2)$ using measurements and predictions

Let's follow this process for inclusive NC DIS measurement...

Selecting NC DIS

Signature of Neutral Current Deep Inelastic Scattering at HERA

A high energy scattered beam electron balanced in transverse momentum (P_T) by the hadronic system

There are 2 large backgrounds to overcome:

- Beam-gas interactions - can be rejected using timing
- fake electrons in photoproduction (γp) events

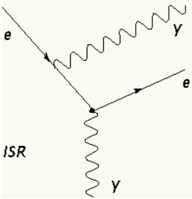
γp is removed by cutting on $\delta = \sum_i (E_i - E_i \cos\theta) = \sum_i (E - p_z)_i$

In γp the electron escapes undetected. In events where the electron is fully reconstructed δ peaks close to $2E^e_{\mathrm{beam}}$.
Once DIS sample selected corrections are applied for Born level cross section

Corrections

- Two types of radiative corrections:
  - Virtual corrections
  - Infrared part of real photon emission

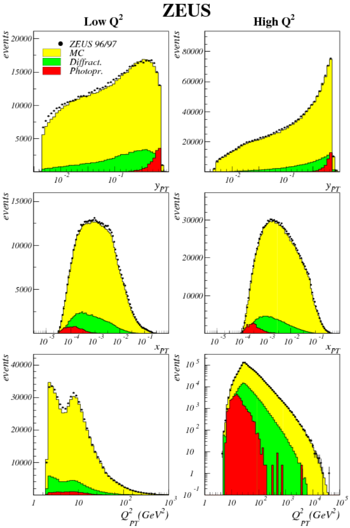
  Necessary because it shifts the true Q^2 and δ from the *true* value, effect is less than 10% at HERA
- Acceptance corrections
  - Factor to get from binned number of events to cross section



Acceptance Corrections

- DIS MC events are generated
- Events are passed through detector simulation
- It is checked that MC decribes key variables
- Bins of x, Q^2 are chosen appropriate to detector resolution
- In each bin 3 indicators are used to establish suitability: Acceptance, Purity, Efficiency

Control Plots



- Description is generally satisfactory
- Especially good at High Q^2

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Purity, Efficiency & Acceptance

Acceptance

The ratio of the number of events measured in a (N_{MC}) to the number of events generated in a bin (N_{True}).

Purity

The ratio of the number of events measured & generated in a bin to the number of events measured in a bin.

Efficiency

The ratio of the number of events measured & generated in a bin to the number of events generated in a bin.

$$A = E/P$$

Bin-by-Bin Unfolding

Cross section measurement

$$\Delta x \Delta Q^2 \frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{N(x,Q^2)}{\langle\mathcal{L}\rangle A(x,Q^2)}$$

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{N(x,Q^2)}{N_{MC}}\left[\frac{N_{true}}{\langle\mathcal{L}\rangle \Delta x \Delta Q^2}\right]$$

$$\left.\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2}\right|^{\mathrm{meas}} = \frac{N_{\mathrm{data}} - N_{\mathrm{bkgnd}}}{N_{MC}} \left.\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2}\right|^{\mathrm{theory}}$$

$$F_2^{\mathrm{em}} = \frac{Q^4 x Y_+}{2\pi\alpha^2} \frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} [1 + \delta_{RC} + \delta_{F_L} + \delta_Z]$$

- δ_{RC},δ_{F_L} and δ_Z are corrections for radiative effects, the longitudinal structure function and Z^0 exchange.

Usually calculation is iterative using

$$F^{i+1}(x, Q^2) = \frac{N_{\mathrm{data}} - N_{\mathrm{bkgnd}}}{N_{\mathrm{MC}}(x, Q^2)} F^i(x, Q^2)$$

Uncertainties

- Experimental and theoretical uncertainties:
  - Luminosity measurement: precision and calibration of the detector, effects from beam satellite bunches → 2.3%
  - Detector simulation: uncertainties in CAL energy scale and in simulation of the CAL and CTD response to $e^{\pm}$ → 4.4%
  - Electroweak parameters: Relevant parameters have been measured to high accuracy & contribute small uncertainty in predicted σ over the HERA kinematic range → 0.25%
  - Radiative corrections: corrections due to ISR convoluted with experimental resolution produce uncertainties $\rightarrow < 2\%$
  - Structure functions:
    - experimental uncertainties $\rightarrow \pm 6.2\%$
    - Uncertainty of the quark-gluon coupling α_S used in the evolution to higher Q^2 → 1.9%
    - total $\pm 6.5\%$
- Total systematic uncertainty $\rightarrow \pm 8.4\%$

Charged Current DIS

- Lowest order EW cross section for the reaction $e^+p \rightarrow \bar{\nu}_e X$:

$$\frac{\mathrm{d}^2\sigma^{\mathrm{CC}}(e^+p)}{\mathrm{d}x\mathrm{d}Q^2} = \frac{G_F^2}{4\pi x}\left(\frac{M_W^2}{M_W^2+Q^2}\right)^2 \{Y_+F_2^{\mathrm{CC}}(x, Q^2) - Y_- xF_3^{\mathrm{CC}}(x, Q^2) - y^2F_L^{\mathrm{CC}}(x, Q^2)\}$$

  with G_F the fermi constant and M_W the W mass

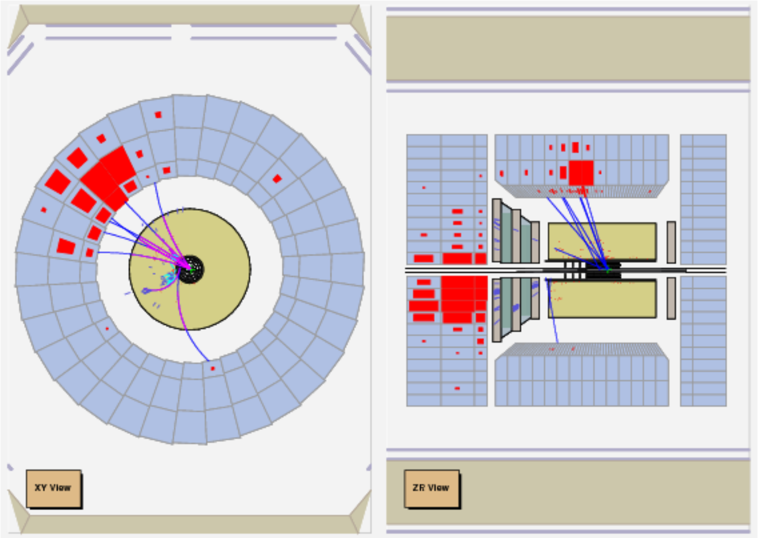
- At LO QCD the structure functions F_2^{CC} and xF_3^{CC} measure sums and differences of quark and antiquark parton momentum distributions

- For longitudinally unpolarised beams:

$$\begin{aligned} F_2^{\mathrm{CC}} &= x[d(x, Q^2) + s(x, Q^2) + \bar{u}(x, Q^2) + \bar{c}(x, Q^2)] \\ xF_3^{\mathrm{CC}} &= x[d(x, Q^2) + s(x, Q^2) - \bar{u}(x, Q^2) - \bar{c}(x, Q^2)] \end{aligned}$$

- F_L^{CC} is zero at LO but finite at NLO and has a negligible contribution to the cross section except at high values of y (close to 1), where it can be as large as 10%

Charged Current Event



PDF Extraction I

- pQCD only predicts the Q^2 evolution of the PDFs, not the x dependence
- Ideally find analytic parametrisations of the PDFs which are consistent with Q^2 dependence predicted by QCD.
- Most common method: perform direct numerical integration of the DGLAP equations at Next-to-leading-order (NLO)

PDF Extraction II

Simple recipe for extracting PDFs:

- Assume different analytic shapes for PDFs (valence, sea, gluon) at some starting scale $Q^2 = Q_0^2$
  - Q_0^2 is arbitary, but must be large enough for $\alpha_{s(Q_0^2)}$ to be small
- Use the DGLAP equations to evolve the PDFs up to a different Q^2 value and use to predict structure functions
- Fit to data

Necessary parameters are those needed to specify the analytic shapes of the PDFs, Λ_{QCD} and $\alpha(M_Z^2)$

- Can use these fits to determine α_S as well as the PDFs

PDF Extraction III

A typical choice of PDFs to fit are:

$$u_v, d_v, S, g, \bar{d} - \bar{u}$$

Usual Form of PDFs

$$\begin{aligned} xu_v &= A_u x^{\lambda_u}(1-x)^{\eta_u} P(x,u) \\ xd_v &= A_d x^{\lambda_d}(1-x)^{\eta_d} P(x,d) \\ xS &= A_S x^{\lambda_S}(1-x)^{\eta_S} P(x,S) \\ xg &= A_g x^{\lambda_g}(1-x)^{\eta_g} P(x,g) \end{aligned} \tag{4}$$

P(x,i) are polynomials in x or $\sqrt{x}$

Not all normalisations A_i are free parameters: A_u, A_d & A_g are constrained by different sum rules

Flavour composition of the sea

- Heavy quarks require special treatment; assume either
  - entirely generated by gluon distribution via $\gamma^* g \rightarrow q\bar{q}(Q^2 \sim m^2_{c,b})$
  - Heavy quark distribution only above threshold $Q^2 \gg m^2_{c,b})$
- Strange quarks suppressed wrt to u & d (larger mass)

$$\bar{s} = \frac{(\bar{u}+\bar{d})}{4}$$

- Historically assume u, d content of sea is symmetric
- no special reason why this should be true
- in fact it seems that $\bar{d} > \bar{u}$

PDF Constraints I

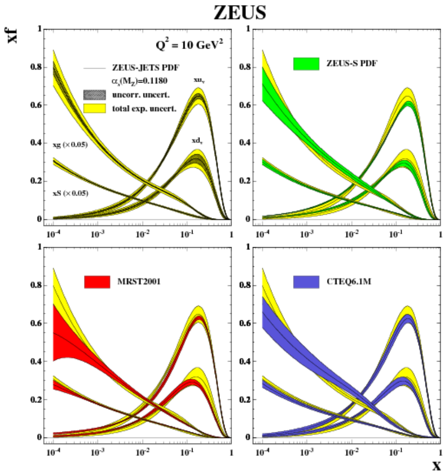
- CCFR neutrino data (xF_3)
  - Valence shapes for all x with u_V & d_V contributing early
  - Most reliable at medium x (worry about nuclear corrections at highest and lowest x)
- NMC data on $\frac{F_2(\mu D)}{F_2(\mu p)}$
  - gives ratio d_V/u_V at large x
  - Only dataset to do so
- $F_2(lD)$ & $F_2(lp)$ from NMC, BDCMS, E665, SLAC and F_2 from CCFR
  - Singlet combination of quarks ($x\Sigma = xu_v + xd_v + xS$)
  - Sea distribution for all (x, Q^2) covered by experiments

PDF Constraints II

Where do different constraints come from?

- F_2 data from the same experiments
    - Combinations of u_v and d_V at high x
    - Contributions weighted by $(\text{quark charge})^2$ (u_v) dominant for protons
    - equal contribution for deuterons
    - u_v better determined than d_v
- F_2 data also constrains gluon density
- CCFR dimuon data
    - Strange Quark distribution
    - directly or cia weak decay of charm quarks
- HERA data
    - Sea quark and gluon distributions

Results of QCD Fits



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- Results from different groups MRST & CTEQ - professional fitters and market leaders
- In these global fits other data, aside from structure function data is also used

Non-DIS PDF Data I

Constraints on quark distributions can also come from:

- Drell-Yan dilepton production: $pN \longrightarrow \mu^+\mu^-X$
  - A sensitive probe of sea quark distribution
  - dominant subprocess: $q\bar{q} \longrightarrow \gamma^* \longrightarrow \mu^+\mu^-$
  - Data from E605 and more recently E772 (moderate to high x)
- Ratio of data $pn \longrightarrow \mu^+\mu^-X$ to $pp \longrightarrow \mu^+\mu^-X$
  - Give information on ration $\frac{\bar{d}}{\bar{u}}$ (NA51 & E866) experiments
- $W^\pm$ production:

$$p\bar{p} \longrightarrow W^\pm X$$

  - Dominant subprocesses: $u\bar{d} \longrightarrow W^+$, $d\bar{u} \longrightarrow W^-$
  - $W^\pm$ asymmetry also gives information on d/u

Non-DIS PDF Data II

- Constraints on the gluon distribution
  - DIS structure function data only really constrains low-x gluon
  - use prompt photon or single inclusive jet production to get high-x gluon
- Prompt Photon data: $pN \rightarrow \gamma X (0.02 < x < 0.5)$
  - Dominant subprocess: $gg - \gamma q$ at leading order
  - Data from WA70, UA6, E706, ISR, UA2, CDF
- High E_T jet production from HERA TeVatron
  - Depend on the gluon via gg, gq and $g\bar{q}$ initiated processes

- DIS offers a precise way of measuring the structure of the proton
- The structure of the proton can be understood in terms of parton distribution functions, described by QCD
- The precision of these data will especially the high x gluon will strongly affect early LHC physics results