QCD Physics - Lecture 3

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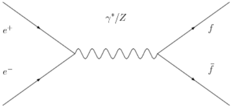
Outline

1. QCD in e^+e^- annihilation
2. Jets in e^+e^- annihilation
3. Event Shapes
4. Triple Gluon Vertex
5. QCD at Future e^+e^- colliders
6. Photon Structure

- Many basic ideas and properties of pQCD can be illustrated by considering hadron production in e^-e^+ collisions
- Total cross section for $e^+e^- \rightarrow$ *hadrons* is one of few quantities for which the first 3 terms in pQCD are known
- pQCD predicts a rich 'jet' structure for the final state hadrons in e^+e^-

Total Hadronic Cross Section



For massless quarks, at energies below the Z pole we have

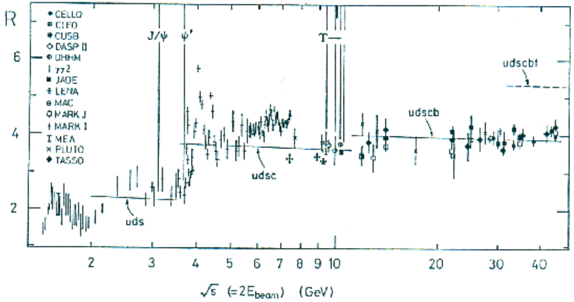
$$R = \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \frac{\sum_q \sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

On the Z pole

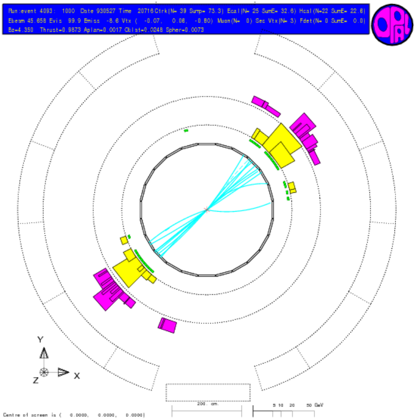
$$R_Z = \frac{\Gamma(Z \to \mathrm{hadrons})}{\Gamma(Z \to \mu^+\mu^-)} = \frac{\sum_q \Gamma(Z \to q\bar{q})}{\Gamma(Z \to \mu^+\mu^-)} = \frac{N_c \sum_q (A_q^2 + V_q^2)}{A_\mu^2 + V_\mu^2}$$

with $q = u, d, s, c, b, N_c = 3$ and $sin\theta_W = 0.23$
$R = 11/3 = 3.67$ and $R_z = 20.09$
Inclusive cross sections have been measured

Numbers of Quark Species



R_z measured a LEP $= 20.79 \pm 0.04$ (3.5% higher than LO). Need NLO correction of factor $\{1 + \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_S^2)\}$, Can be accomodated with $\alpha_S \approx 0.11$
(See Ellis, Stirling & Webber)



- what about the kinematic distribution of hadrons in the final state?
- If hadronic fragments of fast moving quarks have limited transverse momentum relative to the quark momentum (collinear fragmentation), then the lowest order contribution to $\sigma(e^+e^- \rightarrow q\bar{q})$ can be interpreted as production of back to back jets

Jet definition

Quantitive measurements need quantitive jet definition
Some combination of energy flow objects will be used to form a jet, the kinematic properties of the jet object is defined in terms of these energy flow objects (i) as:

Snowmass Convention Jet Kinematic Quantities:

$$E_T^{\text{jet}} = \sum_i E_T^i,\ \eta^{\text{jet}} = \frac{\sum_i \eta^i E_T^i}{E_T^{\text{jet}}}$$

$$\phi^{\text{jet}} = \frac{\sum_i \phi^i E_T^i}{E_T^{\text{jet}}}$$

The pseudorapidity η is defined as $\eta = -\log\tan(\frac{\theta}{2})$
But how do we decide which energy flow objects are assigned to which jets?

Jet Algorithms

Grouping of energy flow objects is performed by jet algorithms

3 popular jet algorithms will be discussed:

- The JADE algorithm
- The Cone algorithm
- The K_T or "Durham" algorithm

In the case of experimental results the energy flow objects used are final state hadrons, usually measured in calorimeter or tracking components

Hadronisation studies show that only small corrections are needed to compare experimental data to parton level calculations.

Kinematics of jet production correspond closely to the kinematics of partons in the hard scatter ("Local hadron-Parton Duality " - LHPD)

Jade Algorithm

The Jade algorithm groups particles to form the minimum invariant mass

It has one parameter y_s

For n energy flow objects

Follow the following steps

- Find pair of objects with minimum invariant mass
- if invariant mass $< y_s$ combine objects
- repeat for $n-1$ objects
- when lowest pair have mass $> y_s$, remaining objects are jets

Jade Algorithm Pros and Cons

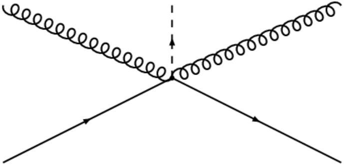
Pros:

- Infra-red safe - emission of a soft/collinear gluon does not affect multiplicity

Cons:

- Not particularly amenable to theoreticians

Example of bad case, 2 soft gluons emitted, can be classified as 3 jet by JADE algorithm:



K_T Algorithm I

The K_T algorithm is an improved version of the Jade algorithm
Follow the following steps

- Find pair
  of objects with minimum value of relative transverse momentum

$$K_T = 2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij}) > y_{\mathrm{cut}}$$

- if relative $K_T < y_{\mathrm{cut}}$ combine objects
- repeat for $n - 1$ objects
- when lowest pair have $K_T > y_s$, remaining objects are jets

This is the algorithm of choice at HERA, where it is formulated in a slightly different way.

K_T Algorithm II

- For every pair of objects, a distance parameter d_{ij} is defined
$$d_{ij} = \min(E_i^2, E_j^2) R_{ij}^2 / R^2$$
with the radius parameter R being of order unity, and R_{ij} the distance between the two objects in the $\eta - \phi$ plane
$R_{ij}^2 = (\Delta\eta_{ij})^2 + (\Delta\phi_{ij})^2$.
- For every single object the distance parameter to the beam is defined as:
$$d_i = E_{T,i}^2$$
- d_i and d_{ij} are compared; if d_{ij} is smaller than d_i then objects i and j are merged according to the Snowmass convention. Otherwise, the object is considered complete and removed from further clustering.
- The previous 3 steps are repeated on the remaining (combined) objects, until $d_i < d_{ij}, \vee\ i$

Cone Algorithm

- Preclusters are identified using a grid in the $\eta - \phi$ plane with a cell spacing of $\Delta\eta^{\mathrm{gridcell}} \approx \; \Delta\phi^{\mathrm{gridcell}} \approx R/2$ where R is the cone radius. Preclusters are defined by moving a 3×3 cell window around the grid. If the total transverse energy within the window is greater than a minimum energy requirement (1 GeV) then it is identified as a precluster.
- A cone of radius R_{cone} is placed around the precluster, and objects within the cone are assigned to the cluster. Typically the cone radius is $R_{\mathrm{cone}} = 1$.
- The kinematics of the cluster are determined according to the snowmass convention
- the previous two steps are repeated until convergence, or a maximum number of iterations is reached.
- Clusters are considered complete jets, if they pass a minimum transverse energy requirement.

Simulation

Our measurements of jet processes will be dependent on our understanding of developement of the final state from the underlying process

There are several stages in the simulation:

- An event generator generates partons → parton level
- A parton shower or cascade is generated using the Sudakov form factor or alternatively via colour dipole splitting
- Some sort of hadronisation model such as the Lund String Model or a cluster model evolves the partons through to final state hadrons (πs etc.) → hadron level
- Final state hadrons pass through the detector simulation,trigger simulation and reconstruction → Detector Level

Monte Carlo has to solve the following problem:
Given the virtual mass scale and momentum fraction (t_1, x_1) after some step of evolution, or as initial conditions, generate the values $(t_2.x_2)$ after the next step
Use

Sudakov Form Factor

Probability of evolving from t_1 to t_2 without resolvable branching is

$$\frac{\Delta(t_2)}{\Delta(t_1)}$$

$\Delta(t)$ is the Sudakov form factor, calculable from GLAP equations. Depends on cut-off, natural resolution limit is t_0 (parton virtual mass squared)

Monte Carlo solves

$$\mathcal{R} = \frac{\Delta(t_2)}{\Delta(t_1)}$$

where $\mathcal{R}$ is a random number distributed uniformly in $[0, 1]$
If t_2 is higher than the hard subprocess scale Q^2 then no more branching occurs. otherwise the value of the momentum fraction $z = x_2/x_1$ is generated for the next branching (Which is done using the GLAP equations)

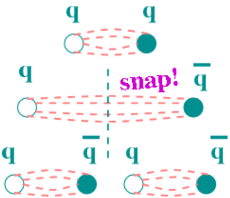
Hadronisation Models

After parton showering/branching we are left with a set of partons with virtualities of the order of the cut-off scale t_0. Now we enter the long distance regime in which non-perturbative effects become important.
The most important of these effects is hadronisation. We only have models to represent this process.
The most popular models are

- Lund String Model - Used in PYTHIA LEPTO,
- Cluster Model - Used in HERWIG

Lund String Model

The Lund String model is conceptually simple:



- As $q\bar{q}$ pairs move apart the colour electric field is compressed to a linear vortex line → The Lund String
- When sufficient energy is stored in a string it can break to form a new $q'\bar{q}'$ pair and so on
- Colour dipole dynamics and radiation is considered

Cluster Model

Preconfinement

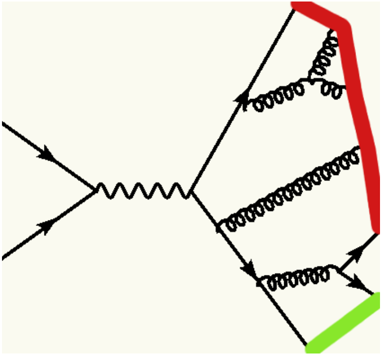
Colour connected neighbouring partons have an asymptotic mass distribution that falls rapidly at high masses and is asymptotically Q^2 independent and universal

Suggests a class of cluster hadronisation models:

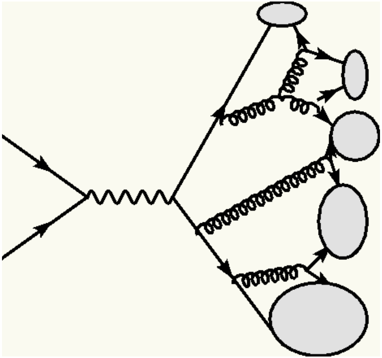
- Colour-singlet clusters of partons are formed after perturbative stage of jet development & decay into observed hadrons
- gluons are split non-perturbatively into $q\bar{q}$ pairs
- neighbouring quarks and antiquarks are combined into singlets
- (HERWIG) Clusters decay isotropically into pairs of hadrons
- (HERWIG) Branching ratios from density of states

String vs Cluster Formation

String



Cluster



Experimentally we measure cross sections at the hadron level.

$$\sigma_{\rm meas} \propto N_{\rm data} \frac{N_{\rm had}^{\rm NC}}{N_{det}^{\rm MC}}$$

According to LHPD we expect these hadron level quantities to correspond closely to the parton level quantitites
In each kinematic bin , the quantity $\delta_{\rm had}$ should be close to unity

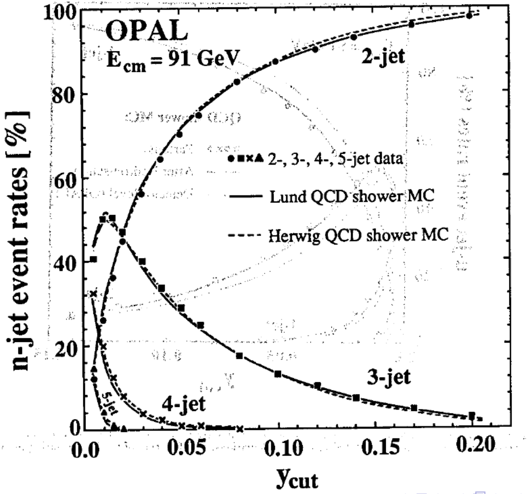
$$\delta_{\rm had} = \frac{N_{part}^{MC}}{N_{had}^{MC}}$$

In comparison with NLO calculations for which no generators are available parton level we compare to $\sigma_{NLO \otimes HAD}$

$$\sigma_{NLO \otimes HAD} = \frac{\sigma NLO}{\delta had}$$

Jet Rates at LEP

Relative rates of jet production were well simulated at LEP



Another way to study jet-like characteristics of hadronic final states is to use 'shape variables'

- Quantities are defined which characterise some aspect of the 'shape' of an event
- The differential cross sections can then be measured and compared to theoretical predictions
- Quantities should be infra-red safe such as those using linear sums of momenta

Thrust

$$\mathcal{T} = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i |\mathbf{p}_i|}$$

$\mathbf{n}$ is an arbitary unit vector $\mathbf{p}_i$ are the final state hadrons

Thrust measures how 'pencil-like' the event is

Spherocity

$$\mathcal{S} = \left(\frac{4}{\pi}\right)^2 \min_{\mathbf{n}} \left(\frac{\sum_i |\mathbf{p}_i \wedge \mathbf{n}|}{\sum_i |\mathbf{p}_i|}\right)^2$$

$\mathbf{n}$ is an arbitary unit vector $\mathbf{p}_i$ are the final state hadrons

Spherocity measures how 'spherical' the event is

$\mathcal{C}$-parameter

$\mathcal{C}$-Parameter

$$\mathcal{C} = \frac{3}{2}\frac{\sum_{i,j}[|\mathbf{p}_i||\mathbf{p}_j| - (\mathbf{p}_i \cdot \mathbf{p}_j)^2/|\mathbf{p}_i||\mathbf{p}_j|]}{(\sum_i |\mathbf{p}_i|^2)}$$

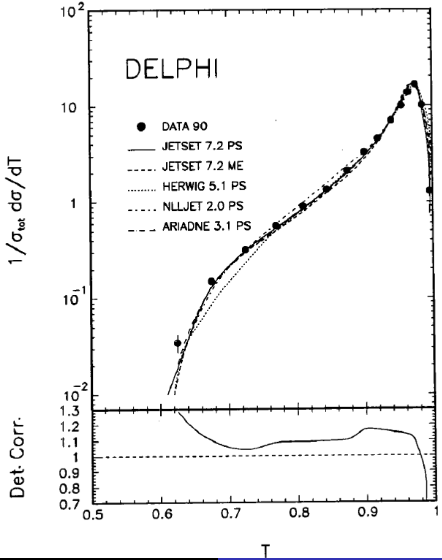
$\mathbf{p}_i, j$ are the final state hadrons

The $\mathcal{C}$-parameter gives an indication of the two-particle correlations in the event

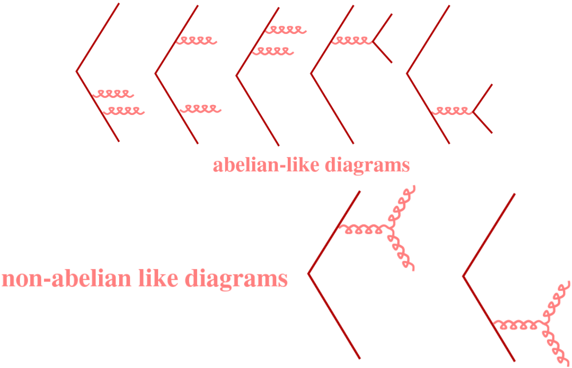
Event Shape Values

Quantity	$\mathcal{T}$	$\mathcal{S}$	$\mathcal{C}$
Pencil like event	1	0	0
Spherical event	$\frac{1}{2}$	1	1
$q\bar{q}g$	$\max\{x_i\}$	$\frac{16}{\pi^2}\frac{\prod_i(1-x_i)}{\max\{x_i^2\}}$	$6\,\frac{\prod_i(1-x_i)}{x_1x_2x_3}$

LEP measurements



The four jet rate at LEP is especially interesting because it's the lowest order cross section in e^+e^- which is sensitive to the triple-gluon vertex



- We already saw that the three jet rate distribution was described by QCD
- It's possible to construct an 'Abelian QCD' theory in which $SU(3)$ is replaced by $[U(1)]^3$, and the coupling is adjusted to $\bar{\alpha_s} = C_F \alpha_S$ so that the correct 3-jet rate is obtained
- An elegant way to distinguish between these models is to use correlations among the final-state particles induced by the different contributions to the cross section
- Several event shape variables have been suggested, the idea being to measure the relative orientation of the planes containing the primary $q\bar{q}$ and secondary (gg or $q\bar{q}$) pairs of jets

θ_{NR}

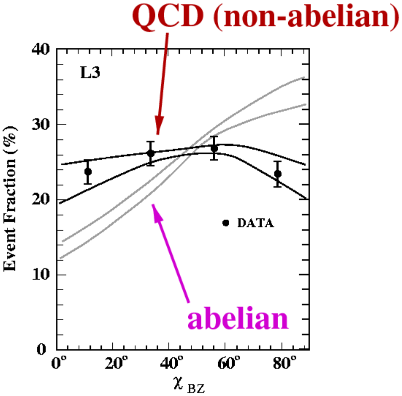
Nachtmann-Reiter Angle

$$\cos\theta_{\mathrm{NR}} = \frac{(\mathbf{p}_1-\mathbf{p}_3)\cdot(\mathbf{p}_2-\mathbf{p}_4)}{|\mathbf{p}_1-\mathbf{p}_3||\mathbf{p}_2-\mathbf{p}_4|}$$

The Nachtmann-Reiter angle θ_{NR}, is the angle between the momentum vector of the differences of jets 1,2 and jets 3,4 (ordered in energy)
There also exists the modified Nachtmann-Reiter Angle (θ^*_{NR})

$$\cos\theta^*_{\mathrm{NR}} = \frac{(\mathbf{p}_1-\mathbf{p}_2)\cdot(\mathbf{p}_3-\mathbf{p}_4)}{|\mathbf{p}_1-\mathbf{p}_2||\mathbf{p}_3-\mathbf{p}_4|}$$

χ_{BZ}

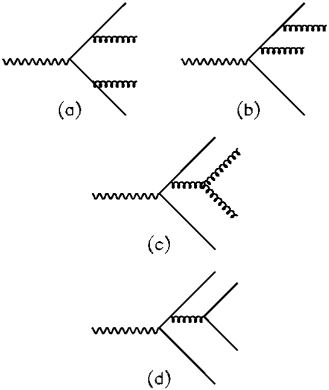


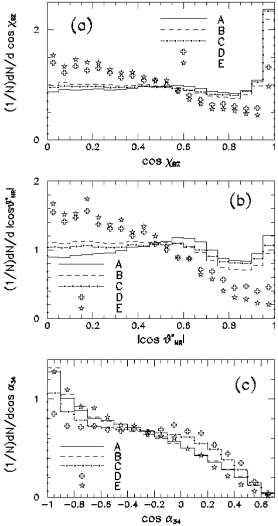
Bengtsson-Zerwas Angle

$$\chi_{\mathrm{BZ}} = \frac{(\mathbf{p}_1 \wedge \mathbf{p}_2) \cdot (\mathbf{p}_3 \wedge \mathbf{p}_4)}{|\mathbf{p}_1||\mathbf{p}_2||\mathbf{p}_3||\mathbf{p}_4|}$$

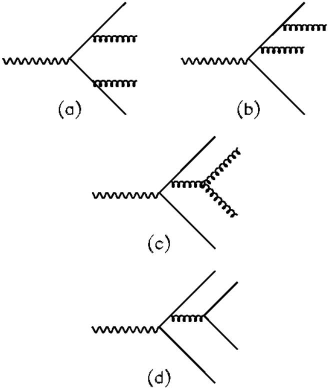
The Bengtsson-Zerwas Angle is the angle between the planes containing the two highest and two lowest energy jets

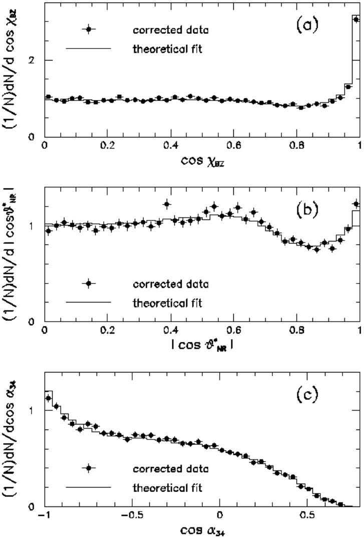
Distributions of variables





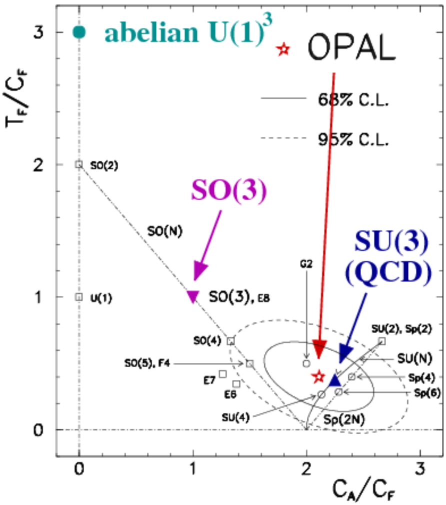
Distributions of variables





Fit can be used to extract Colour Factors

Colour Factors



- Predictions:

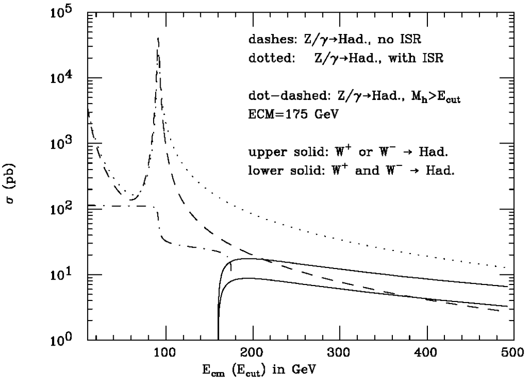
Group	C_A/C_F	T_F/C_F
$SU(3)$	$9/4$	$3/8$
$[U(1)]^3$	0	3
$SO(3)$	1	1

- Measured values are:

$$C_A/C_F = 2.11 \pm 0.16(\text{St.}) \pm 0.28\ (\text{Sy.})$$
$$T_F/C_F = 0.40 \pm 0.11(\text{St.}) \pm 0.14\ (\text{St.})$$

- SU(3) is clearly favoured by fit

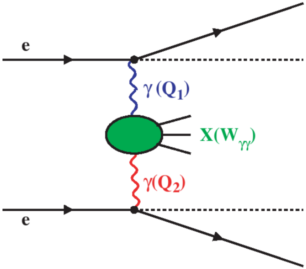
- At SLC and LEP corrections due to hadronisation are small → Jets give direct insight into the production and properties of q and g
- Precision on measurements of $\alpha_{\mathrm{S}}(M_Z^2)$ is of order 5%
- Success is partly due to the peak hadronic cross section being large (peaks on Z-pole)
- Above the Z peak the cross section falls rapidly then tend to $\propto 1/s$
- Therefore Future high-energy e^+e^- colliders would find it difficult to match the precision of the QCD studies from direct e^+e^- anihilation
- There are other processes that can contribute



- $e^+e^- \rightarrow W^+W^- \rightarrow$ jets
- ISR events where photons are radiated from the beams causing $\sqrt{s}$ to move towards the Z pole
- Two photon induced hadronic events

The latter process is especially interesting in the context of photon structure

Photons can oscillate into $q\bar{q}$ pairs. 2-photon processeses at LEP



Kinematics

Photon Virtuality:

$$Q_1^2 = -q_1^2 = 2E_1E_1'(1 - cos\theta_1)$$
$$Q_2^2 = -q_2^2 = 2E_2E_2'(1 - cos\theta_2)$$
$$W_{\gamma\gamma}\text{: } \gamma\gamma \text{ center-of-mass energy}$$
$$W_{\mathrm{vis}}^2 = (\textstyle\sum_h E_h)^2 - (\sum_h \mathbf{p}_h) \leq W_{\gamma\gamma}^2$$

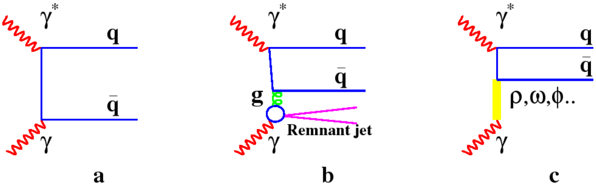
h = hadronic particle measured in detector

$Q^2 \equiv Q_1^2 > Q_2^2 \equiv P^2 \approx 0$ Single Tag

$$x = Q^2/2(p.q) = Q^2/Q^2 + W_{\gamma\gamma} + P^2$$
$$y = (q.p)/(k.p) = 1 - (E_{\mathrm{tag}}/E_{\mathrm{beam}})\cos^2\theta_{\mathrm{tag}} \approx 0$$

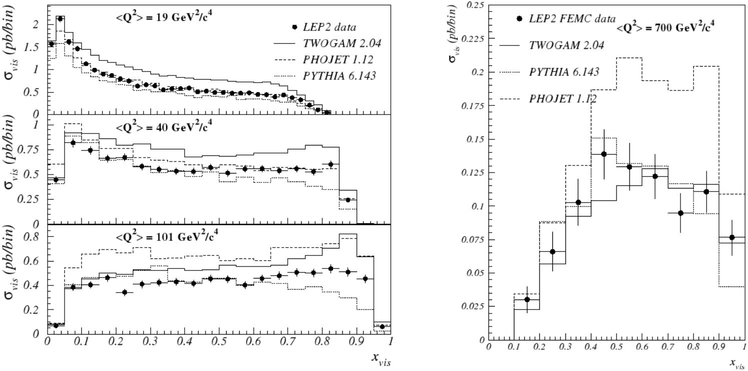
F_2^γ

$$F_2^\gamma = x \sum_{i=1}^{N_f} e_i^2 (q_i^\gamma + \bar{q}_i^\gamma)(x, Q^2)$$



- (a) Pointlike coupling → Fully calculable in QED
- (b) QCD corrections → DGLAP equations
- (c) Initial conditions → non perturbative (VDM) contribution

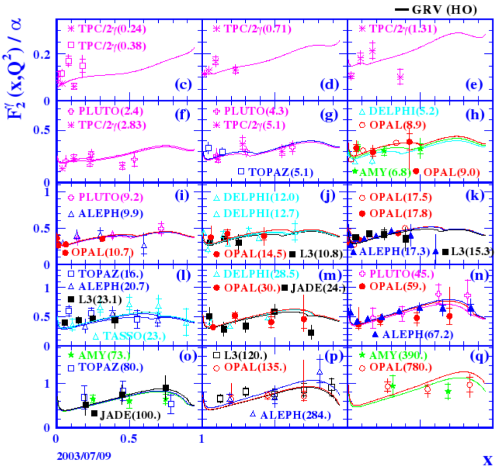
F_2^γ II



No MC model reproduces distributions for all Q^2

MC model dispersion is often the dominant systematic error

F_2^γ II



Precision significantly worse than proton F_2

Summary/ Conclusions

- The clean environment of e^+e^- collisons has yielded a rich set of QCD results
- QCD + hadronisation models can describe Jet and event structure
- Photon structure has been measured in e^+e^-, the precision is not so good as measurements of proton structure