QCD Physics - Lecture 4

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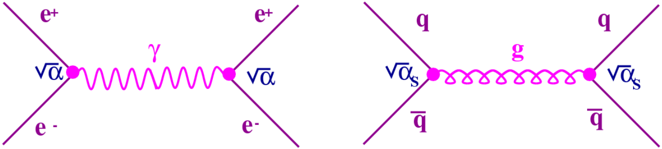
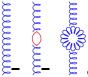
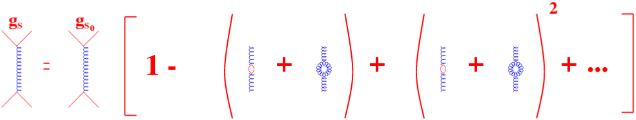
6^{th} February 2007

Outline



- α is a measure of the strength of the EM interaction.$\alpha = \frac{e^2}{4\pi}$
- In QED, α gives the probability of emitting a photon
- a factor $\sqrt{\alpha}$ is associated with each absorption or emission of a photon by a charge e
- α_S is the QCD analogue to α: $\alpha_S = \frac{g_S^2}{4\pi}$ where g_S is the colour charge
- In QCD, α_S gives the probability of emitting a gluon.
- a factor $\sqrt{\alpha_S}$ is associated with each absorption or emission of a gluon by a colour charge g_S

- The magnitude of the colour charge is modified by antiscreening
- The gluon propagator  is modified to  at leading order
- Summing to all orders gives:



- The sum of a geometric series giving $g_S^2 = g_{S_0}^2 \left(\frac{1}{1+I(Q^2)}\right)$
- We expect that α_S will run with Q^2

- In QCD, $I(Q^2) = \frac{\alpha_S(\mu^2)}{12\pi}(33 - 2N_F)\ln\frac{Q^2}{\mu^2}$, μ, a reference scale
- At sufficiently low Q^2, the effective coupling becomes large
  The scale at which this happens is
  $\Lambda^2 = \mu^2 \exp\left[\frac{-12\pi}{(33-2N_F)\alpha_S(\mu^2)}\right]$
- Then: $\alpha_S(Q^2) = \frac{12\pi}{(33-2N_F)\ln\frac{Q^2}{\mu^2}}$
- $\alpha_S(Q^2)$ decreases with increasing Q^2 and is small for short distance interactions
- for $Q^2 \gg \Lambda^2$, perturbative description in terms of quarks and gluons interacting weakly is possible
- for $Q^2 \approx \Lambda^2$, the quarks and gluons are tightly bound in hadronic states and perturbation theory is inapplicable
- The important parameter Λ is not predicted by theory and must be determined by experiment. Recent measurements give $\Lambda = 208^{+25}_{-23}$ MeV

Measuring α_S

- α_S is one of the fundamental parameters of QCD
- The value of α_S is not predicted by theory
- To obtain a high precision, the combination of many measurements must be performed to reduce the uncertainties
- Measurements as precise as possible: using phase space where pQCD predictions least affected by theoretical uncertainties
- α_S has been measured in NC DIS using different methods:
  - Dijet cross sections in the Breit Frame at high Q^2
  - Inclusive jet cross sections in the Breit frame at high Q^2
  - Internal structure of jets: jet shape
  - Internal structure of jets: subjet multiplicity
  - NLO QCD combined fit to inclusive NC DIS measurements (F_2) to obtain simultaneously the pPDFs and α_S
- Independent methods give results consistent with each other and the world average

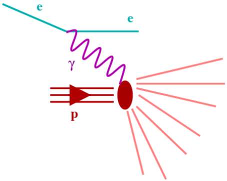
- Method to determine α_S is usually to make a measurement which is sensitive to it and perform a fit with α_S as the free parameter
- In recent years, much progress on the developement of process and observable independent algorithms that allow complete and analytical cancellation of soft and collinear singularities encountered in the calculation of the NLO cross sections
- Programs computing arbitary infrared/collinear-safe DIS observables in NLO QCD exist (DISENT, MEPJET, DISASTER++, NLOJET).
- Restricting the measurements to a high-Q^2 kinematic region:
  - Avoids large renormalisation-scale dependence of NLO QCD dijet σs at low Q^2 (20-50 % for $10 < Q^2 < 100\ \mathrm{GeV}^2$)
  - Reduces uncertainty due to the pPDFs (esp. gluon density)
  - Improves the reconstruction of the boost in the Breit frame
- Asymmetric cuts on E_T (jets) avoid infrared sensitive regions where behaviour of σ predicted by NLO QCD programs unphysical

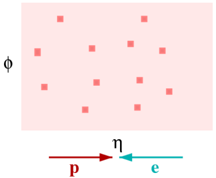
γp at HERA

- *ep* scattering is dominated by γp interactions in which a quasi-real γ ($Q^2 \approx 0$) emitted by *e* interacts with a parton from the proton
- Total γp cross section has been measured at HERA:

$$\sigma_{\gamma p}^{\mathrm{tot}} = 143 \pm 4(\mathrm{stat}) \pm 17(\mathrm{syst})\mu\mathrm{b}$$

- Most of the γp cross section is due to soft processes

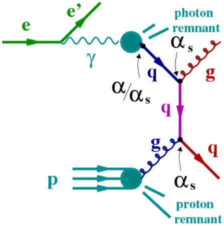




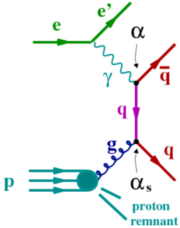
- However at large γp CMS energies available at HERA, $100 < W_{\gamma p} < 300$ GeV, a fraction of the γp interactions are expected to produce high transverse energy jets
- Main source of jets at HERA is hard scattering in γp interactions.

- photon at $Q^2 \approx 0$ can couple directly in a $\gamma q\bar{q}$ vertex (normal QED gauge boson behaviour).
- Or it can fluctuate into an intermediate vector meson[1] state which interacts via its partonic structure.

A) Resolved:



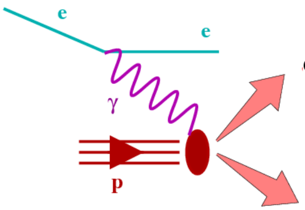
b) Direct:

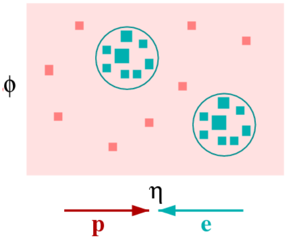


- Resolved coupling allows study of parton content of photon.
- Direct coupling allows study of parton content of proton.

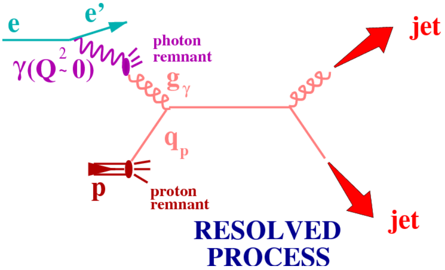
[1] Sakurai Phys. Rev. Lett. 22, 981 (1969)

- Evidence for hard photon scattering has been observed in $\gamma\gamma$ interactions in e^+e^-
- CMS Energy was too low to observe jet production.
- At HERA the CMS energies available provide a wider phase space, enabling the observation of hard scattering in photoproduction.





- Observation of resolved processes:



- The gluon from the proton carries only a fraction of the photon's energy:

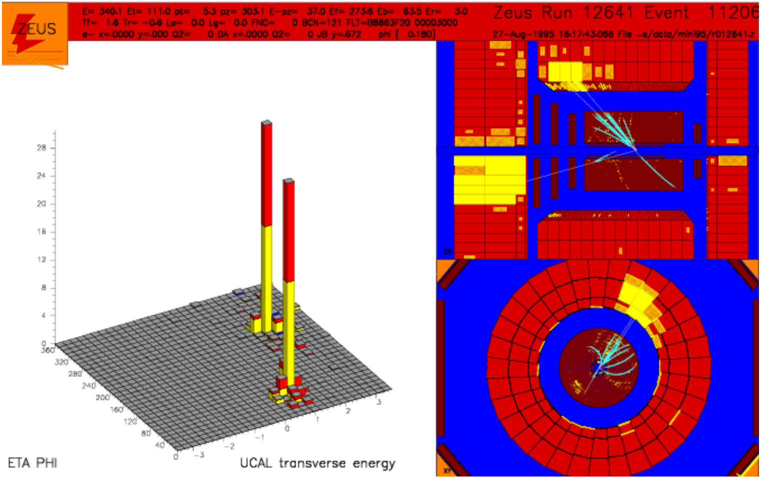
$$P_{g\gamma} = x_\gamma p_\gamma$$

- The photon energy is not invested fully in the hard collision:

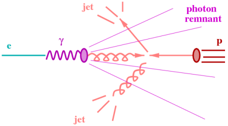
$$x_\gamma < 1$$

- Outgoing jets are boosted in the proton direction.
- a hadronic photon remnant is expected in the electron direction.

resolved γp Event

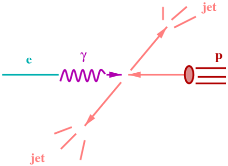


Direct Events



Resolved:

- Photon remnant.
- $x_\gamma \ll 1$



Direct:

- No photon remnant.
- All γ energy in interaction $x_\gamma \to 1$

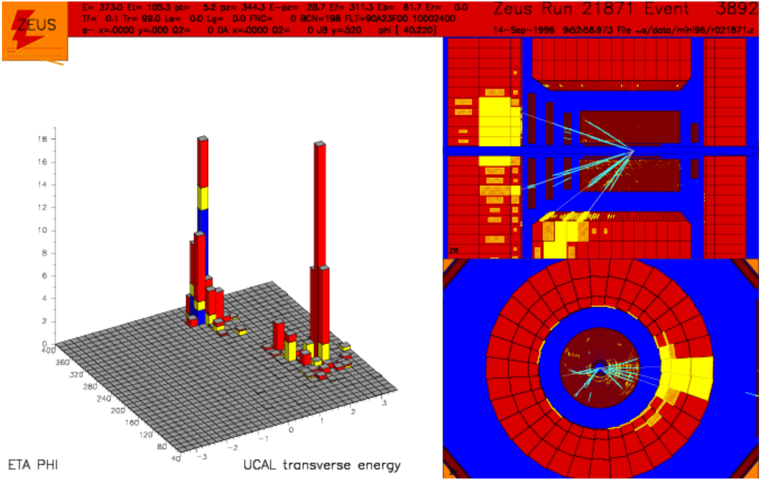
$$x_\gamma p_\gamma + x_p p_p = p_L^{\text{jet1}} + p_L^{\text{jet2}} \quad (1)$$

$$-x_\gamma p_\gamma + x_p p_p = E^{\text{jet1}} + E^{\text{jet2}} \quad (2)$$

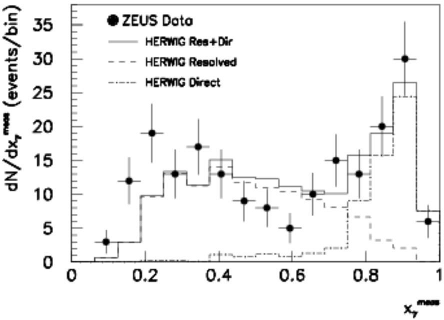
$$p_\gamma = yE_e \quad (3)$$

$$\rightarrow x_\gamma^{\text{obs}} = \frac{1}{2yE_e}\left(E^{\text{jet1}} e^{-\eta^{\text{jet1}}} + E^{\text{jet2}} e^{-\eta^{\text{jet2}}}\right)$$

Direct γp Event

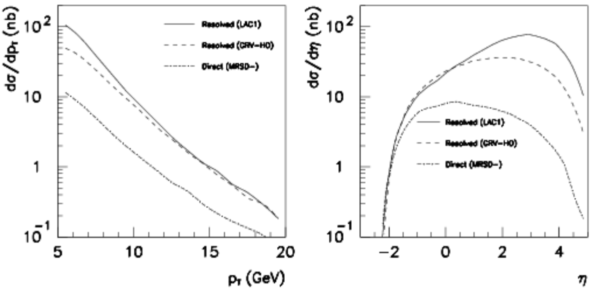


$$x_\gamma^{\text{obs}} = \frac{1}{2yE_e}\left(E^{\text{jet1}} e^{-\eta^{\text{jet1}}} + E^{\text{jet2}} e^{-\eta^{\text{jet2}}}\right)$$

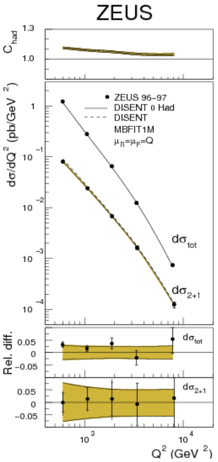


First observation of dijet structure in direct photoproduction data

Jet Cross Sections at LO

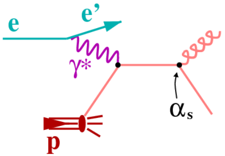


- resolved processes dominate for a wide range of p_T.
- Direct processes are significant only in the tails
- η for resolved processes boosted in the proton direction
- η distribution for direct processes more central.



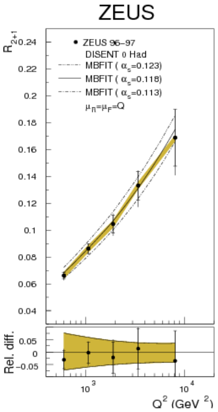
$$ep \rightarrow e + \text{jet} + \text{jet (exclusive)}$$

- Breit frame $\rightarrow$ Q has vector $(0,0,0,Q)$, struck q enters with (0,0,0,$q/2$), rebounds as hitting a brick wall
- $470 < Q^2 < 20000\ \mathrm{GeV}^2$
- $E_{T,1}^{BRE} > 8$ GeV, $E_{T,2}^{BRE} > 5$ GeV, $-1 < \eta^{LAB_{1,2}} < 2$



$$\mathrm{d}\sigma_{ep\rightarrow e\ \text{jet jet}} = \sum \mathrm{d}x f(x,\alpha_S)\mathrm{d}\sigma_{\gamma^* q\rightarrow qg}(x,\alpha_S)$$

- High Q^2 means exp. and theory errors very low
- Shape,magnitude of σ_{meas} described by NLO



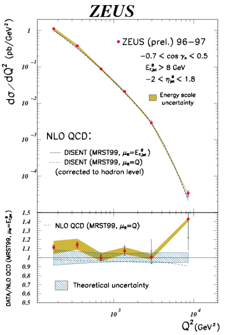
- Extraction of α_S is made from ratios of observables to reduce uncertainties & dependence on PDFs

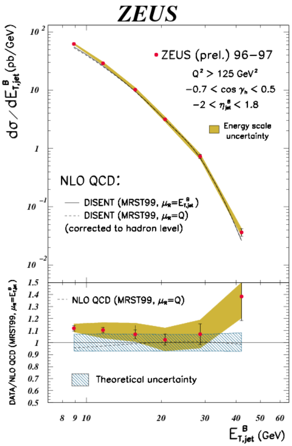
$$\rightarrow \text{Ratio } \mathrm{R}_{2+1} \equiv \frac{\mathrm{d}\sigma_{2+1}/\mathrm{d}Q^2}{\mathrm{d}\sigma_{\mathrm{tot}}/\mathrm{d}Q^2}$$

- Small experimental uncertainties
- Small theoretical uncertainties:
  - Higher-order terms ($> NLO$) ($\sim 5\%$)
  - Value of α_S assumed ($\sim 6\%$)
  - Uncertainties on the proton PDFs ($\sim 1.5\%$)
  - Hadronisation corrections ($< 10\%$)
- Comparison with NLO QCD calculations:
  - Measured ratio is described by prediction, demonstrating validity of the description of dynamics of dijet production by NLO QCD hard processes

$$ep \to e + \text{ jet} + X$$

- Kinematic region $Q^2 > 125\ \mathrm{GeV}^2$ and $-0.7 < \cos\theta < 0.5$ where γ corresponds to the direction of the scattered quark in the QPM
- ≥ 1 jet with $E^B_{T,\mathrm{jet}} > 8$ GeV and $-2\eta^B_{\mathrm{jet}} < 1.8$.
- Pros of inclusive jet cross sections in a QCD analysis:
  - Infrared insensitivity (not divergent when $E_g \to 0$)
  - For dijet cross section, assymetric cuts on $E^B_{T,\mathrm{jet}}$ are necessary to avoid the infrared-sensitive regions where NLO QCD programs are not reliable.
- Better to test resummed calculation
- Smaller theory uncertainties than in dijet σ



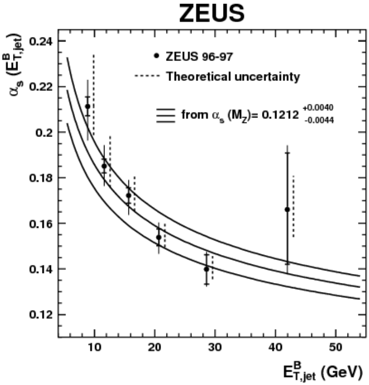


$$ep \longrightarrow e + \text{ jet} + X$$

Comparison with NLO QCD calculations:

- the measured inclusive jet cross sections are well described by the predictions at high Q^2 and at high $E^B_{T,\text{jet}}$.
- At low Q^2 and at low $E^B_{T,\text{jet}}$, the measurements of inclusive jet cross sections are above the calculations by $\sim 12\%$ (origin of discrepancy at present unknown).
- Therefore, the determination of α_S from these measurements is restricted to high scales

- The QCD prediction for the energy-scale dependence of α_{S} from the measured differential cross sections at different scales $\rightarrow$ from the measured $\frac{d\sigma}{dE^{\mathrm{jet}}_{T,B}}$ in each $E^{\mathrm{jet}}_{T,B}$ region, $\alpha_{\mathrm{S}}(E^{\mathrm{jet}}_{T,B})$

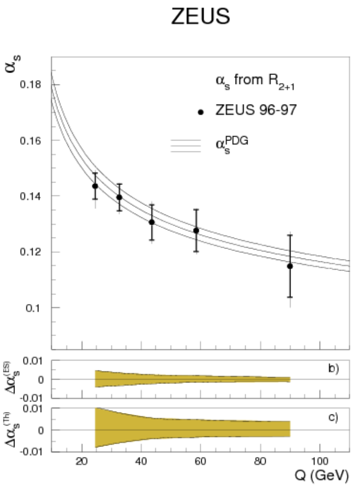


- The procedure to determine $\alpha_S(M_Z^2)$ from the dijet fraction as a function of Q^2, $R_{2+1}(Q^2)$ was:
  - NLO calculations were performed using three sets of the MBFIT pPDFs and the value of $\alpha_S(M_Z^2)$ assumed in each calculation is that of the PDF set.
  - These were used to parameterise the $\alpha_S(M_Z^2)$ dependence of $R_{2+1}(Q^2)$, according to:

$$R_2^i(\alpha_S(M_Z^2)) = A_1^i \alpha_S(M_Z^2) + A_2^i \alpha_S^2(M_Z^2)$$

  - The value of α_S was then determine by a χ^2 fit to the parameterisation of the measured values.
- this procedure correctly handles the complete α_S dependence on the NLO cross sections (explicit dependence on partonic σ implicit from pPDFs) and preserves the correlation between α_S and the PDFs

α_S From Dijets



Study the scale dependence of $\alpha_S(Q)$:

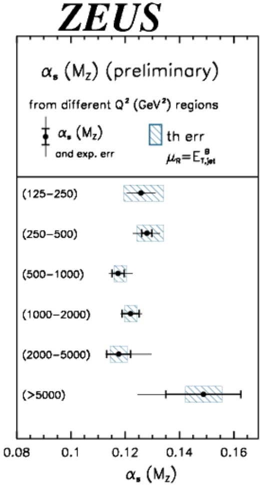
- α_S extracted from $R_{2+1}(Q^2)$ in each Q^2 region
- Results consistent with running of α_S QCD

Combined value of α_S extracted:

$$\alpha_S(M_Z^2) = 0.1166 \pm 0.0019(\text{stat.})\ ^{+0.0033}_{-0.0024}(\text{exp.})\ ^{+0.0044}_{-0.0057}(\text{th.})$$

Theoretical uncertainties dominate:

- Terms beyond NLO
- Uncertainties from pPDFs
- Hadronisation corrections
- Need improvement in theoretical calculations to obtain a more precise determination of α_S from the dijet cross section at high Q^2



Similar method to dijet measurement:

- Differential σs are used not dijet fraction
- Different values obtained.

$\mathrm{d}\sigma/\mathrm{d}Q^2$, $Q^2 > 125\ \mathrm{GeV}^2$.

$$\alpha_S(M_Z^2) = 0.1241 \pm 0.0009(\mathrm{stat.})\ ^{+0.0038}_{-0.0043}(\mathrm{exp.})\ ^{+0.0036}_{-0.0052}(\mathrm{th.})$$

$\mathrm{d}\sigma/\mathrm{d}Q^2$, $Q^2 > 500\ \mathrm{GeV}^2$.

$$\alpha_S(M_Z^2) = 0.1190 \pm 0.0017(\mathrm{stat.})\ ^{+0.0023}_{-0.0049}(\mathrm{exp.})\ ^{+0.0026}_{-0.0028}(\mathrm{th.})$$

$\mathrm{d}\sigma/\mathrm{d}E_T^{\mathrm{jet}}$, $E_T^{\mathrm{jet}} > 14\ \mathrm{GeV}$.

$$\alpha_S(M_Z^2) = 0.1206 \pm 0.0015(\mathrm{stat.})\ ^{+0.0045}_{-0.0058}(\mathrm{exp.})\ ^{+0.0039}_{-0.0041}(\mathrm{th.})$$

Need improvement in theoretical calculations to obtain a more precise determination of α_S from inclusive cross sections.

The procedure to determine $\alpha_S(M_Z^2)$ from the inclusive jet cross section as a function of $E_T^{\rm jet}$ is as follows:

- QCD predictions for inclusive the cross sections use:

$$\frac{d\sigma}{dE_T^{\rm jet}} = \alpha_S^2(\mu_R)\hat{X}^{(0)}(\mu_F, E_T^{\rm jet})[1 + \alpha_S(\mu_R)k_1(\mu_R, \mu_F, E_T^{\rm jet})]$$

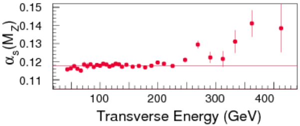
  Where $\alpha_S^2(\mu_R)\hat{X}^{(0)}(\mu_F, E_T^{\rm jet})$ is the LO prediction for the inclusive jet cross section and $\alpha_S^3(\mu_R)\hat{X}^{(0)}(\mu_F, E_T^{\rm jet})k_1(\mu_R, \mu_F, E_T^{\rm jet})$ is the NLO contribution
- Both $\hat{X}^{(0)}(\mu_F, E_T^{\rm jet})$ and $k_1(\mu_R, \mu_F, E_T^{\rm jet})$ are calculated using the JETRAD program
- The CTEQ4M pPDF sets were used
- Values of α_S were then determined by comparing the theory to the measurements

Extracted Values

From the measured $\frac{d\sigma}{dE_T^{\text{jet}}}$, a value of $\alpha_S(M_Z^2)$ was extracted for each E_T^{jet} region and a combined value of

$$\alpha_S(M_Z^2) = 0.1178 \pm 0.0001(\text{stat.})\ {}^{+0.0081}_{-0.0095}(\text{exp.})\ {}^{+0.0092}_{-0.0075}(th.)$$

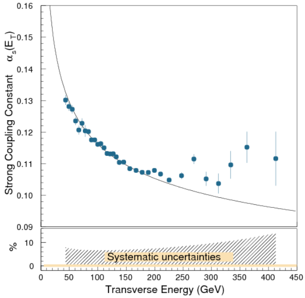
was obtained for $E_T^{\text{jet}} < 250$ GeV



- For $E_T^{\text{jet}} < 250$ GeV, good agreement with the world average
- Behaviour at high E_T^{jet} is a direct reflection of the excess observed in $d\sigma/dE_T^{\text{jet}}$ $\rightarrow$ This discrepancy may be explained by an enhanced gluon content at high x in the pPDFs
- Experimental and theoretical uncertainties are of the same order, both need to be improved to improve measurement

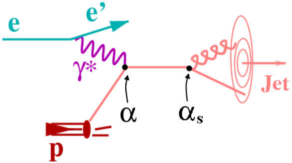
Energy Scale Dependence

The QCD prediction for the energy-scale dependence of α_S was tested using the measured $\frac{d\sigma}{dE_T^{\text{jet}}}$ at different E_T^{jet} values



The results are in good agreement with the predicted running of α_S over a large range in E_T^{jet} $(40 < E_T^{\text{jet}} < 250\text{ GeV})$

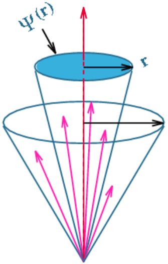
- Internal jet structure yields independent method to extract α_S
- Integrated Jet Shape and Mean Subjet Multiplicity in inclusive jet NC DIS are calculable in pQCD at high E_T^{jet}
- Dependence of calculations on knowledge of pPDFs reduced
- Lowest non-trivial-order contribution to measurements from $\mathcal{O}(\alpha\alpha_S)$ pQCD calculations.
- Measuring jet substructure provides a stringent test of pQCD beyond LO and allow determination of α_S by comparing NLO calculations of substructure to measurements, since in the Lab frame it's possible to have 3 partons inside one jet



Integrated Jet Shape

Integrated jet shape is defined as the average fraction of the jet E_T lying inside a cone in the $\eta-\phi$ plane of radius $r=\sqrt{(\Delta\eta)^2+(\Delta\phi)^2}$ concentric with the jet axis.

$$\langle\psi(r)\rangle = \frac{1}{N_{\text{jets}}}\sum_{\text{jets}}\frac{E_t(r)}{E_T^{\text{jets}}}$$

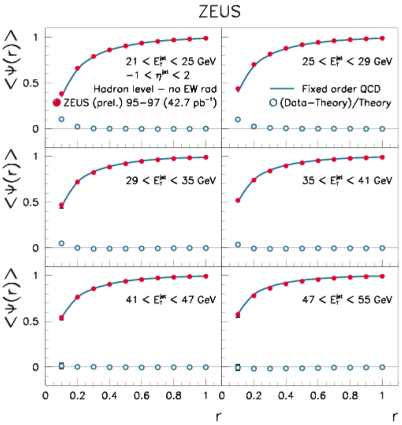


In pQCD $\langle 1-\psi(r)\rangle$ is calculated; the fraction of the jet's E_T due to parton emission lying in cone segment between r and $R=1$

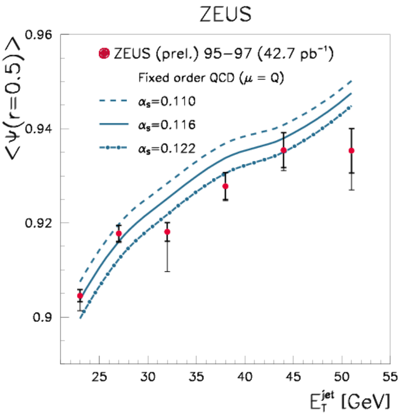
$$\langle 1-\psi(r)\rangle = \frac{\int \mathrm{d}E_T E_T[\mathrm{d}\sigma(ep\to 2\text{partons})/\mathrm{d}E_T]}{E_T^{\text{jet}}\sigma_{\text{jet}}(E_T^{\text{jet}})}$$

where $\sigma_{\text{jet}}(E_T^{\text{jet}})$ is the σ for inclusive jet production.
NLO QCD predictions for $\langle\psi(r)\rangle$ are derived from the formula by computing numerator to $\mathcal{O}(\alpha\alpha_S^2)$ and denominator to $(\mathcal{O}(\alpha\alpha_S))$

$$ep \longrightarrow e + \text{jet} + X$$



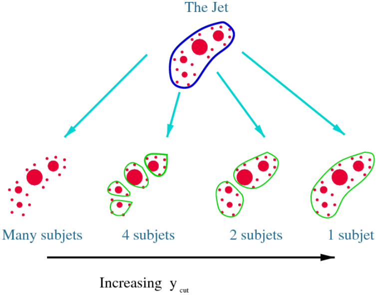
- Small exp. uncertainties ($r \geq 0.5$)
  - Fragmentation model dependence: $< 0.8\%$
  - Jet energy scale: $< 0.4\%$
- Small corrections s for $r \geq 0.5$.
  - Data (detector effects): $< 3\%$
  - NLO QCD calculations $< 5\%$
- Comparison with NLO QCD:
  - Calculations provide a very good description of the data: (Data-NLO)/NLO smaller than 1.3 for $r = 0.5$



- Integrated jet shape at fixed values of r, $< \psi(r = 0.5) >$, increases as E_T^{jet} increases: the jets become more collimated, $\rightarrow$ effect of running of α_S.
  - Measurements are sensitive to α_S
- Sensitivity of $< \psi(r = 0.5) >$ as a function of E_T^{jet} to the value of $\alpha_S(M_Z^2)$ seen by comparing measurement to NLO QCD using three different values of $\alpha_S(M_Z^2)$
- Calculations provide very good description of the data $\rightarrow$ method can be used to extract $\alpha_S(M_Z^2)$

Subjet Multiplicity

- Subjets resolved by reapplying the k_T algorithm on all the particles belonging to the jet until, for every pair of particles the $d_{ij} > d_{\rm cut} = y_{\rm cut}(E_T^{\rm jet})^2$
- Remaining clusters are called subjets
- Subjet structure depends on $y_{\rm cut}$



- In pQCD the mean subjet multiplicity, $\langle n_{\rm subjet}\rangle$ is calculated as the ratio of σs for $n_{\rm subjet} - 1$ to inclusive jet production

$$\langle n_{{\rm sbj}(y_{\rm cut})}\rangle = 1 + \frac{1}{\sigma_{\rm jet}} \sum_{j=2}^{\infty} (j-1) \cdot \sigma_{\rm sbj}(y_{\rm cut})$$

- NLO QCD predictions for the mean subjet multiplicity are derived from this formula by adding 1 and computing the numerator to $\mathcal{O}(\alpha\alpha_S^2)$ and the denominator to $\mathcal{O}(\alpha\alpha_S)$

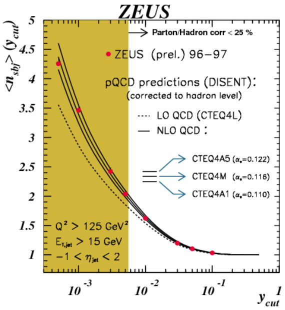
Subjet Multiplicity II

Theoretical advantages of $\langle n_{\text{subjet}} \rangle$

- "safe" (definable at any order in pQCD)
- useful tool to investigate colour dynamics.
- small hadronisation corrections

Small uncertainties for $y_{\text{cut}} \geq 0.01$

- Fragmentation model dependence: $< 3\%$
  jet energy scale: $\sim 1\%$

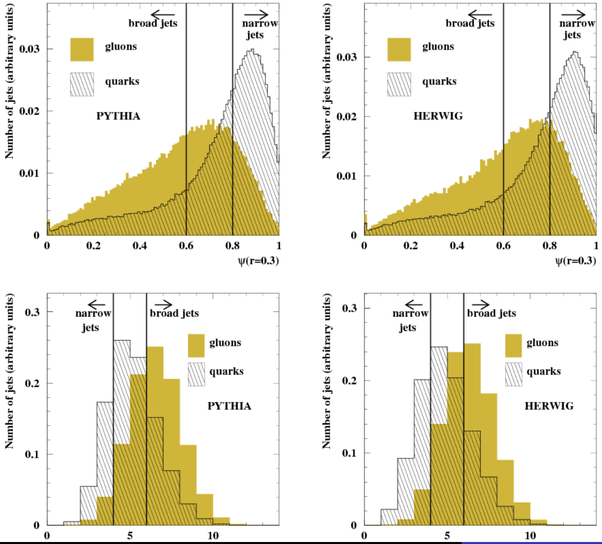


Small corrections for $y_{\text{cut}} \geq 0.01$

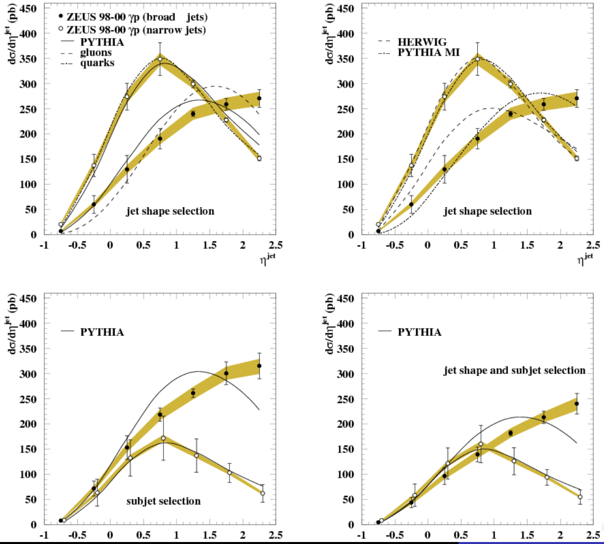
- (Detector effects): $< 10\%$
  NLO(parton-hadron effects): $< 15\%$ for $E_T^{\text{jet}} > 25$ GeV.

Comparison with QCD calculations:

- the LO calculations fail to describe the data.
- NLO calculations provide a good description.



- Gluon jets should be broader than Quark jets
- $\psi(r = 0.3)$ peaks at lower values for gluons than for quarks
- $n_{\text{subjet}}(y_{\text{cut}} = 5.10^{-4})$ peaks at higher values for gluons than for quarks



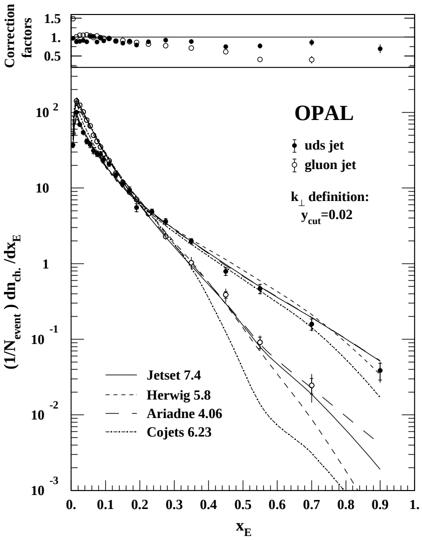
- Quark and gluon MC normalised to data cross sections describe narrow and broad jets respectively

Aside: q and g jets

There are characterisitc differences between jets originating from quarks and gluons:

- Average multiplicity of any type of object in a gluon jet should be $C_A/C_F = 9/4$ times greater than that in a quark jet
- As a result of higher multiplicity, gluon jets have a softer fragmentation function - The highest energy particle in a quark jet has a higher proportion of the jet energy that in a gluon jet
- Jets are also broader as shown in previous slides.

These differences arise from the higher effective colour charge-squared of the gluon (proprtional to C_A).



Charged multiplicity distribution at LEP

- Peaks are in the same place
- Gluon peak is higher by approx C_F/C_A
- Gluon fragmentation relatively reduced at high x

- Procedure to determine $\alpha_S(M_Z^2)$ from the measured $< \psi(r = 0.5) >$ for $E_T^{\text{jet}} > 21$ GeV and $< n_{\text{subjet}} >$ at $y_{\text{cut}} = 0.01$ for $E_T^{\text{jet}} > 25$ GeV is the same as before.
  - NLO calculations were performed using five sets of CTEQ4 pPDFS and the value of $\alpha_S(M_Z^2)$ assumed in each calculation is that of each PDF set.
  - Calculations are used to parametrise the $\alpha_S(M_Z^2)$ dependence of $V(< \psi(r = 0.5) >)$, $< n_{\text{subjet}} >$ at $y_{\text{cut}} = 0.01$
- The value of $\alpha_S(M_Z^2)$ was then determined by a χ^2 fit of the parameterisation of the measured values.
- This procedure correctly handles the complete α_S dependence on the NLO cross sections (explicit from dependence on partonic σ implicit from pPDFs) and preserves the correlation between α_S and the PDFs.

- From the measured $< \psi(r = 0.5) >$ and $< n_{\mathrm{subjet}} >$ at $y_{\mathrm{cut}} = 0.01$ in each E_T^{jet} region a value of α_S has been extracted from each observable.

Measurement from ψ

$$\alpha_S(M_Z^2) = 0.1179 \pm 0.0014(\mathrm{stat.})\ ^{+0.0065}_{-0.0054}(\mathrm{exp.})\ ^{+0.0073}_{-0.0094}(\mathrm{th.})$$

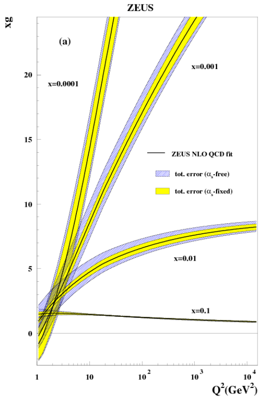
Measurement from n_{subjet}

$$\alpha_S(M_Z^2) = 0.1185 \pm 0.0016(\mathrm{stat.})\ ^{+0.0048}_{-0.0067}(\mathrm{exp.})\ ^{+0.0071}_{-0.0089}(\mathrm{th.})$$

- The theoretical uncertainties dominate:
  - Terms beyond NLO
  - Hadronisation corrections.
  - Need improvement in theoretical calculations to obtain a more precise determination of α_S from the internal structure of jets.

α_S From NLO Fit

- In the evolution of singlet q distributions at intermediate x $(0.01 < x < 0.3)$, value of $\alpha_S(M_Z^2)$ and g density are strongly correlated (DGLAP equations) & an increase in $\alpha_S(M_Z^2)$ can be compensated by a harder g distribution
- At small $x(x < 0.01)$ correlation is weakened, since the g then drives the behaviour of F_2 as well as $\mathrm{d}F_2/\mathrm{d}\ln(Q^2)$
- Precision low-x data can be used in a simultaneous fit of $\alpha_S(M_Z^2)$ & PDFs
- The difference between the PDF parameters obtained in this way compared to the result obtained using a fixed value of $\alpha_S(M_Z^2)$ is neglible, but the uncertainties are a bit larger.



Value of α_S extracted from fit

$\alpha_S(M_Z^2) = 0.1166 \pm 0.0008(\text{stat.}) \pm 0.0032(\text{corr.}) \pm 0.0036(\text{norm.}) \pm 0.0018(\text{model})$

Measurement of α_S is based on the ration of hadronic to leptonic decay given theoretically by:

$$R_T = \frac{\Gamma(\tau \to \nu_\tau + \text{hadrons})}{\Gamma(\tau \to \nu_\tau \bar{\nu}_e e)} = 3S_{EW}(1 + \delta_{pQCD} + \delta_{NpQCD})$$

$S_{EW} = 1.0194$ well known from electroweak theory

$\delta_{pQCD} = a + 5.2a^2 + 26.4a^3(\pm 130a^4)$, $a = \frac{\alpha_S(m_\tau^2)}{\pi} \sim 0.1$

$\delta_{nPQCD} = -0.007 \pm 0.004$

and measured from

$$R_\tau = \frac{1 - B_e - B_\mu}{B_l} = \frac{1}{B_l} - 1 - f_\mu$$

B_l lepton branching fraction $f_\mu = 0.9726$ a phase space correction

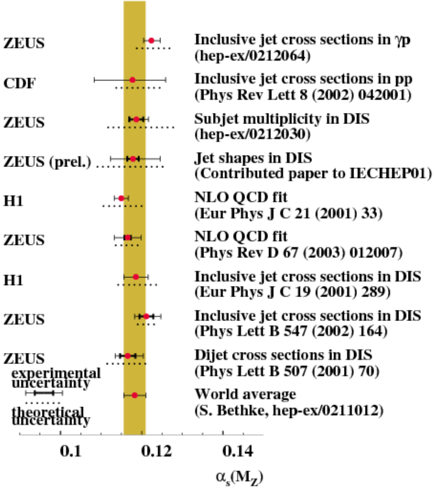
Can be measured from $B_e = B_l$, $B_\mu = f_\mu B_l$ and lifetime $\tau_\tau = B_l \tau_\mu (m_\mu / m_\tau)$

$$\alpha_S(m_\tau^2) = 0.335 \pm 0.021 \text{ (ALEPH)}$$

- Some of the best understood processes in hadron-hadron collisions are vector boson and vector boson + jet production (see next lecture)
- From the ratio of these two cross sections UA1 and UA2 obtained

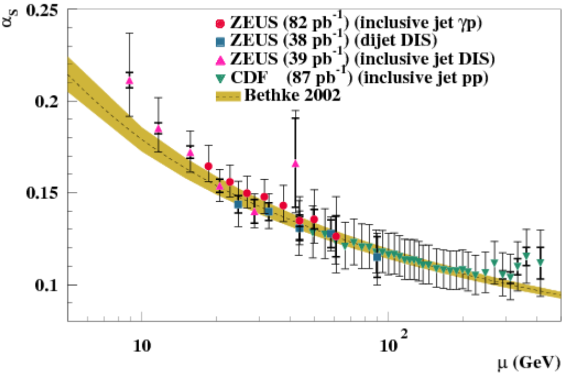
$$\alpha_S(M_W^2) = 0.127 \pm 0.026(\text{stat.}) \ \pm 0.034(\text{sys.}) \ \text{UA1}$$
$$\alpha_S(M_W^2) = 0.123 \pm 0.018(\text{stat.}) \ \pm 0.017(\text{sys.}) \ \text{UA2}$$

Precision much less than other methods



All the measurements are consistent with each other and the world average

Running of α_S



All the measurements are consistent with the running of α_S as predicted by QCD